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Applications of Integration and Differentiation in Real Life

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Abstract: In this paper we will learn about how differentiation and integrations can be applied in real life situations, definite integral, indefinite integral, formulas of integration, characteristics of integration and what is the use of integration in real life.

Keywords: *Differentiation Calculus, Integration Calculus*

1.Introduction:

Integration is employed to address two distinct types of Problems involving the derivative of a function, or its rate of change, are the first type. We know the function, or the slope of its graph, and we wish to find it. As a result, we're required to reverse the differentiating process anti- is the term for this reversal procedure. finding an indefinite integral, differentiation, or finding a primitive function.

The second sort of problem entails adding up a large number of very little amounts (and then reaching a limit as the size of the quantities approaches zero while the number of terms approaches infinity). The definite is defined as a result of this process. integral definite integrals are used to calculate area, volume, gravity centre, and moment of inertia. inertia, force-driven work, and a variety of other applications.

We will explain the history of calculus and the study between them as follows:

- Calculus is calculus in Latin, it studies the continuous math of change in the way geometry is, and the two branches of calculus are integration reflects the process of differentiation.
- Differentiation is concerned with instantaneous changes, and the slopes of curves, while integration is concerned with the accumulation of quantity, and the areas under or between curves.
- The two branches are linked by fundamental theory, and they make use of the basic concepts of convergence of infinite series.
- It studies limit, derivatives, integration, smoothness, and aquatic series, a science that studies and analyses in functions, calculus in uses, plans, sciences, oceanography, oceanography, and problems that algebra simply cannot solve.
- Calculus is usually studied after the basics of algebra, geometry and trigonometry.
- By the conclusion of the seventeenth century, infinite calculus was independently invented.
- Integration between integration and differentiation.
- There is evidence from history that integration is used in some other way to calculate area and volume.
- Beginning with the ancient Egyptians and then the Greeks used integration, then the old method they were using was developed by Archimedes, who introduced the method of exhaustion to the science of integration.
- The scientist Isaac Newton and the scientist Gottfried Leibniz developed the principles invented by Archimedes, and Ibn al-Haytham was able in the Islamic era to develop the science of integration, and then after that the Chinese, after that the science of integration and differentiation was translated and taught to the West and the development of mathematics and physics in which integration and differentiation are involved in a large way.

(1.1) Definition:

Integration is the process of bringing together distinct data. The integral is used to determine the functions that will characterise the area, displacement, and volume of a collection of small data that cannot be measured

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individually. The concept of limit is used in calculus to construct algebra and geometry in a broad sense. Limits

assist us in analysing the outcome of points on a graph, such as how they go closer to each other until their distance is nearly zero. There are two sorts of calculus that we are aware of.

•Differential Calculus

•Integral Calculus

The concept of integration was created to address the following issues:

•When the derivatives of the problem function are known, find it.

•Under specific constraints, find the region limited by the graph of a consists The "Integral Calculus," which includes both definite and indefinite integrals, was born out of these two issues. The Fundamental Theorem of Calculus connects the concepts of differentiating and integrating functions in calculus.

(1.2) Derivative of $f(x)$:

If $f(x)$ is a real function, we define the derivative of $f(x)$ by the limit:

$$L = \lim_{a \to 0} \frac{f(x+a) - f(x)}{a} \qquad (1)$$

Where this limit is exist.

(1.3) Define Integral:

The top and lower bounds are both contained in a definite integral. Only a true line is allowed for x to lie on. The Riemann Integral is another name for the Definite Integral.

The following is the representation of a definite Integral:

$$\int_a^b f(x)dx \qquad (2)$$

(1.4) Indefinite Integral:

Without upper and lower boundaries, indefinite integrals are defined. It's written like this:

$$\int f(x)\ dx = F(x) + c \quad (3)$$

The integrand is the function $f(x)$, where c is any constant.

2.What is the use of Integration in Real Life?

Real-world applications of integrals include engineering, where engineers employ integrals to establish the geometry of a structure. In physics, it's employed to describe the center of gravity among other things. Three-dimensional models are presented in the subject of graphical representation.

(2.1) Example:

Find the Area under the curve of a function $f(x) = 7 - x^2$, where $-1 \leq x \leq 2$

Solution:

$$A = \int_a^b y\, dx \qquad (4)$$

$$A = \int_c^d x\, dy \qquad (5)$$

In this example, we use (4), (i.e. x-axis)

$$A = \int_{-1}^{2} (7 - x^2)dx = 18\ sq.units$$

(2.2) Example:

Find the Area of the region bounded above by: $y = x + 6$, bounded below by:

$y = x^2$ and bounded on the sides by the line $x = 0, x = 2$.

Solution:

$$A = \int_a^b |y_1 - y_2|\, dx \qquad (6)$$

Or:

$$A = \int_c^d |x_1 - x_2|\, dy \qquad (7)$$

In this example, we use (6) (i.e., x-axis)

$$A = \int_0^2 (x + 6 - x^2)dx = \frac{34}{3}\ sq.units$$

(2.3) Example:

Evaluate the area of one left of the rose $r = \sin 3\theta$,between $\theta = 0, \theta = \frac{\pi}{3}$

Solution:

We use this equation.

$$A = \int_a^b \frac{1}{2} r^2 d\theta \qquad (8)$$

Then:

$$A = \int_0^{\frac{\pi}{3}} \frac{1}{2} (\sin 3\theta)^2 d\theta = \frac{\pi}{12}$$

(2.4) Example:

Evaluate the arc length of the curve of the semi-cubical parabola $y^2 = x^3$, between the two point (1,1) and (4,8)

(a) With respect to x:

Solution:

We use:

$$L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\ dx \qquad (9)$$

Then:

$$L = \int_1^4 \sqrt{1 + \frac{9}{4}x}\ dx\ = \frac{1}{27}\,(80\sqrt{10} - 13\sqrt{13})$$

(b) With respect to y :

Solution:

We use:

$$L = \int_c^d \sqrt{1 + (\frac{dx}{dy})^2}\ dy \qquad (10)$$

Then:

$$L = \int_c^d \sqrt{1 + \frac{4}{9}\,(y)^{\frac{-2}{3}}\,dy}\ = \frac{40\sqrt{40}}{27} - \frac{13\sqrt{13}}{27}$$

(2.5) Example:

Find the area of the surface that is generated by rotating the portion of the curve.

$y = 2\sqrt{x}$, $1 \le x \le 2$, about the $x - axis$.

Solution:

$$S = \int_a^b 2\pi y \sqrt{1 + (\frac{dy}{dx})^2}\ dx \qquad (11)$$

$$S = \int_c^d 2\pi x \sqrt{1 + (\frac{dx}{dy})^2}\ dy \qquad (12)$$

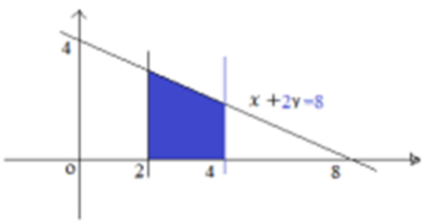
In this case we use (11), (with respect to x), then.

$$S = \int_1^2 2\pi(2\sqrt{x}) \sqrt{1 + (\frac{1}{x})^2}\ dx$$

$$= \frac{8\pi}{3}\left(3\sqrt{3} - 2\sqrt{2}\right) sq.units$$

(2.6) Example:

By using integration to find the area of the shaded region in the figure below about x-axis.



Solution:

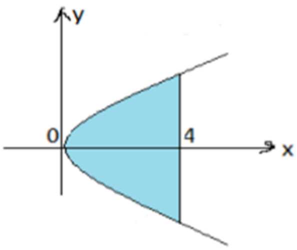
The given line is $x + 2y = 8\ or\ y = \frac{8-x}{2}$, thus the given line is the curve $y = f(x)$, using equation (4)

Area of required region is:

$$A = \int_2^4 \frac{8 - x}{2} dy = 5\ sq.units$$

(2.7) Example:

By using integration find the area of the region bounded by the parabola $y^2 = 16x$ and line $x = 4$



Solution:

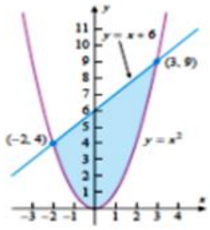
Area of the required region by using equation (4) is:

$$A = 2\int_0^4 y\,dx = 2\int_0^4 4\sqrt{x}\ dx = 8\int_0^4 \sqrt{x}\,dx$$

$$A = \frac{16}{3}\left[x^{\frac{2}{3}}\right]_0^4 = \frac{128}{3} sq.units$$

(2.8) Example:

Find the area of the region that is enclosed between the curves $y = x^2 \ and \ y = x + 6$



Solution:

The graph of the region from figure shows that the lower boundary is $y = x^2$and the upper boundary is $y = x + 6$. We must find the endpoints of the region, the upper and lower boundaries, which have the same y-coordinates, so we equate tow equations $y = x^2 \ and \ y = x + 6$.

These yields

$$x^2 = x + 6$$

$$x^2 - x - 6 = 0$$

$$x = -2, x = 3$$

we get the area:

$$A = \int_{-2}^{3} (x + 6 - x^2) dx$$

$$A = \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right] \begin{matrix} 3 \\ -2 \end{matrix} = \frac{45}{2} \ sq.units$$

There are many applications for integration into one variable, and we will suffice with this, for example, but not exclusively, and we will move in the next paragraph to applications for functions that are in more than one variable.

3.Some Example of Application of Integration on more than one Variable

(3.1) Example:

Calculate the integral if $f(x, y) = xy^2$,where $0 \leq x \leq 2, 0 \leq y \leq 1$.

Solution:

We use this equation.

$$A = \int_a^b (\int_c^d f(x, y) dx) dy \qquad (13)$$

Now we apply in (12)

$$A = \int_0^2 (\int_0^1 (xy^2) dx) dy = \frac{2}{3}$$

(3.2) Example:

Calculate the integral if $f(x, y, z) = kz$,where $0 \leq x \leq 4, 0 \leq y \leq 4, 0 \leq z \leq 4$.

Solution:

We use this equation.

$$A = \int_a^b \int_c^d \int_n^m f(x, y, z) \ dxdydz \quad (14)$$

Now we apply in (14)

$$A = \int_0^4 (\int_0^4 (\int_0^4 kz \ dx) dy)) dz = 128k$$

There is a lot of importance of integration applications in mathematics, and for people in practical life, we will explain as follows:

- It helps analyze systems and find the best solutions to predict the future.
- It helps and plays an important role in practical life, such as solving areas of overlapping shapes.
- Use integration to find the volume of an object with curved sides.
- Integration is also used to find the midpoint of a given area of a solid with two curved sides.
- Integration is used to find the volume of a solid with curved sides.
- Integration is used to calculate the mean of the curve.

4.Some Example of Application of Derivative:

Derivatives have many uses outside of only mathematics and everyday life, including in subjects like science, engineering, physics, and others. You must have mastered the ability to calculate the derivative of a variety of functions in earlier courses, including trigonometric functions, implicit functions, logarithm functions, etc. You will discover how derivatives are used in relation to mathematical ideas and in actual situations in this part.

In general, we can say that:

- Using graphs to determine business profit and loss.
- In order to measure temperature variation.
- To calculate the rate of travel, such as miles per hour or kilometres per hour.

- Several equations in physics are derived using derivatives.
- While studying seismicity, it's interesting to discover the earthquake's magnitude range.

(4.1) Example

Show that the function $y = x^3 - 2x^2 + 2x$, is increasing on Q,where $x \in Q$.

Solution:

$$\frac{dy}{dx} = 3x^2 - 4x + 2 > 0, \forall x$$

This mean that y is increasing function.

(4.2) Example

The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through a point. Find the coordinates of the point.

Solution:

$$\frac{dy}{dx} = 2x - 5 \,, at\ x = x_1 \Rightarrow \frac{dy}{dx} = 2 \Rightarrow 2x_1 = 7 \Rightarrow x_1 = \frac{7}{2}$$

$$y_1 = (\frac{7}{2})^2 - 5\left(\frac{7}{2}\right) + 5 = -\frac{1}{4}$$

$$y + \frac{1}{4} = 2\left(x - \frac{7}{2}\right) \Rightarrow 4y + 1 = 8x - 28 \Rightarrow 8x - 4y - 29 = 0$$

$\Rightarrow x = \frac{1}{3}, y = -7$, satisfies the equation.

5. Some applications that do not need to be prove and we can understand directly

- When you understand information that you did not understand before, this is an example of a function because the field and the corresponding field have changed, and it is an application from real practical life.
- When the car is moving at a speed, this is an application of the differential.
- When a car that was traveling at a certain speed stops, this is an application of integration.
- Calculus and integration are used to solve systems of differential equations that are used in modeling infectious diseases, through which we can predict to determine the type of disease (considered a health application)
- Calculus is used in modelling that conceders them with mechanics and electricity to produce a car that travels the longest possible period by consuming the least petroleum materials for the longest possible period (considered an engineering application).

6. Conclusion:

We have been exposed to a few applications of calculus, for example only, and not exclusively.

We will consider this paper as an introduction to a large series of applications that will see the light soon.

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