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Effects Of MHD and Heat Generation/Absorption on UCM Fluid Across A Porous Stretching Surface

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Abstract: In the current study, the influences of the numerical study ofthermal conductivity in the presence of non-uniform heat sources or sinks and MHD (magnetic field)motivateus to examine the impact of thermal conductivity characteristics of Maxwell fluid over a porous stretching sheet. We delve into various parameter effects through graphical representations.The presence of a transverse magnetic field in an incompressible, electrically conducting fluid causes the velocity field to decrease, causing the temperature field to rise. The results reveal that temperature rises for positive values of the heat source/sink parameters and falls for negative values. Therefore, non-uniform heat sinks are preferable for cooling purposes.

Keywords: *Stretching surface, UCM fluid, Magnetic field, heat generation/absorption, porous medium*.

1 Introduction

Studying flow induced by a stretching surface in a still fluid and subsequent heat transfer is crucial for various technological applications, such as plastic sheet extrusion, wire production, papermaking, crystal growth, and glass blowing. These processes require bringing molten liquid into a cooling system and occasionally stretching it, as in polymer extrusion, to cool it. Achieving the desired fluid properties in such processes is based on cooling rate and stretching rate. Selecting the right cooling liquid is essential as it directly affects the cooling rate, and optimal stretching must be maintained to avoid unwanted property changes. Therefore, The major goal of this study is to have a thorough comprehension of flow heat transfer characteristics.

In the realm of numerous applications similar to the process of polymer extrusion, Crane [1] pioneered the analytical examination of boundary layer flow caused by a stretching sheet. He assumed that the sheet's velocity varied linearly with distance from slit. In the context of the Navier-Stokes equations, Crane's solution for the flow caused by a stretched sheet is an important illustration of an exact solution, and its singularity has long been proved. Many authors have since explored the presence and uniqueness of solutions for stretching sheet-induced flow, as documented by Rajagopal and McLeod [2] and Troy et al [3]. McLeod and Rajagopal's analytical investigation sheds light on the treatment of infinity when solving ordinary differential equationsin bounded domains.

More recently, several researchers have extended Crane's work to encompass Newtonian and non-Newtonian boundary layer flows under differentphysical conditions. When examining heat transfer from an isothermal stretched sheet, Gupta and Gupta [4] took suction/blowing effects into account. Gupta and Gupta's study was developed by Chen and Char [5] to handle non-isothermal stretching sheets with suction and blowing. Using the power law fluctuation of surface temperature, Grubka, and Bobba [6]conducted heat transfer investigations, while Chiam [7] examined magnetohydrodynamic heat transfer from a non-isothermal stretched sheet.

The impact of the heat source/sink phenomenon is a crucial aspect deserving careful attention, as it wields a significant effect on the heat transfer characteristics within exothermic processes. Numerous researchers have explored heat transfer phenomena by examining either a constant heat source/sink or one that varies with temperature, as seen in the works of Rollins and Vajravelu[8]. Furthermore Emad[9] expanded on this by considering the impactof a non-uniform heat source/sink, which varies in both space and temperature, on heat transfer.

Anon-Newtonian 2nd-grade fluid proves inadequate in yielding meaningful outcomes when dealing with highly elastic fluids like polymer melts, especially at high Deborah numbers (See Hayat et al. [10, 11]. Consequently, the importanceof the findings presented in the aforementioned studies is somewhat constrained, particularly in the context of the polymer industry. It is

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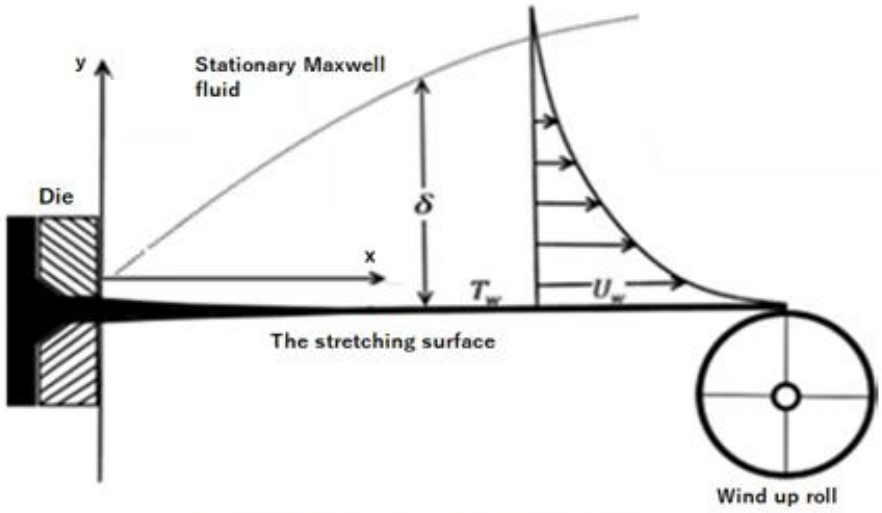
essential to use more realistic viscoelastic fluid models, such as the Upper-Convected Maxwell model or the Oldroyd-B model, in the study to give the theoretical conclusions industrial significance. As mentioned by Sadeghy et al. [12], and Hayat et al. [13] These 2 fluid models were recently applied to examine the viscoelastic fluids flow through both stretching" and non-stretched surfaces. However, heat transfer effects were not taken into account.

Several researchers like Sadeghy[14], Alizadeh-Pahlavan et al. [15, 18], Renardy[16], Rao and Rajagopal [17], Aliakbar et al. [19,20],etc.., have all explored the UCM fluids using the various analytical and numericalmethods with or without considering the heat transfer effects.

However, the influence of thermal conductivity in the incidence of non-uniform heat sources or sinks motivates us to examine the impact of thermal conductivity features of Maxwell fluid across a porous stretching sheet. We delve into various parameter effects through graphical representations.

2 Mathematical Formulation

The flow is caused due to "stretching of a porous sheet", that drags from a thin slit. Two forces of equal magnitude but in opposite directions are applied to the sheet in a way that stretches the wall while keeping its starting point unmovable.



Schematic diagram of the flow problem

The equations governing the flow problem (Refs. Renardy[12],Sadeghy et.al[16], Alizadeh-Pahlavan and Sadeghy[15]) are as mentioned below.

$$u_x + u_y = 0 \quad (1)$$

$$uu_x + vu_y + \lambda\left[u^2u_{xx} + v^2u_{yy} + 2uvu_{xy}\right] = vu_{yy} - \frac{\sigma {B_0}^2}{\rho}u - \frac{\nu}{k\prime}u, \quad (2)$$

$$uT_x + vT_y = \frac{k}{\rho C_p}T_{yy} + \frac{q'''}{\rho C_p} + \frac{\mu}{\rho C_p}{u_y}^2 \quad (3)$$

where "u and $v$indicate the velocity components along x and y directions respectively, $t$denotes the temperature of the fluid, σ presents the density, υrepresents the kinematic viscosity, k' denotes the porosity parameter, C_p signifies the specific heat at constant pressure, $k$indicates the thermal conductivity of the liquid far away from the sheet, B_0, denotes the strength of the magnetic field, υrepresents the inematic viscosity of the fluid andλ denotes the relaxation time Parameter of the fluid. The model of the non-uniform heat source/sink, q''', is as follows: (see Chiam[7])

$$q''' = \frac{ku_w(x)}{x\upsilon}[A*(T_S - T_0)f' + (T - T_0)B*], \quad (4)$$

where the coefficients A* and B* represent the space-dependent heat source/sink and temperature-dependent properties, respectively. Here, we make a note that the scenario in which,$A^* > 0, B^* > 0$ corresponds to the formation of heat on the inside of the system, and that is the case in which, $A^* < 0, B^* < 0$corresponds to the absorption of heat on the inside of the system.

The boundary conditions are,$u = bx, v = 0, T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 PSTCase$

$$-KT_y = Q_w = D\left(\frac{x}{l}\right)^2 PHF\ Case\ at\ y = 0$$

$$u \to 0, u_y \to 0, T \to T_\infty, asy \to \infty \quad (5)$$

HereA and D indicate the constants, b denotes the constantrecognized as the stretching rate, l the

characteristic length, $T_w$signifies the wall temperature and T_∞represents the constant temperature far away from the"sheet.

The dimensionless parameter are;

$$u = bxf_\eta(\eta), v = -\sqrt{b\gamma}f(\eta), Where\eta = \sqrt{\frac{b}{\gamma}}y\theta(\eta)$$
$$= \frac{T - T_\infty}{T_w - T_\infty}, whereT_w - T_\infty$$
$$= A\left(\frac{x}{l}\right)^2 PSTCase$$
$$= D\left(\frac{x}{l}\right)^2 PHFCase$$
(6)

where subscript η denotes derivative w.r.tη. U& v gratify (1). Substituting (6) in (2) and (3)

$$f''' - M^2 f' - (f')^2 + f'' + \beta(2ff'f'' - ff''') + k_2 f' = 0, \quad (7)$$

$$Pr[2f'\theta - \theta' f] = \theta'' + Ec\,Pr\,f''^2 + (A^* f' + B^*\theta), \quad (8)$$

$$Pr[2f'g - g'f] = g'' + Ec\,Pr\,f''^2 + (A^* f' + B^* g), \quad (9)$$

Equation (5)'s boundary conditions change to

PST CASE

$$f_\eta(\eta) = 1,\ \theta(\eta) = 1,\quad f(\eta) = 0\ \ at\ \eta = 0$$
$$f_\eta(\eta) \to 0,\quad \theta(\eta) \to 0,\quad f_{\eta\eta}(\eta) \to 0\ as\ \ \eta \to \infty$$

(10)

PHF CASE

$$f_\eta(\eta) = 1, \theta_\eta(\eta) = -1\quad f(\eta) = 0\ at\ \eta = 0$$
$$f_\eta(\eta) \to 0,\ \theta(\eta) \to 0\quad f_{\eta\eta}(\eta) \to 0\ as\ \eta \to \infty$$

(11)

β denotes the elastic parameter,

k_2 represents "the porosity parameter,

M- presents the magnetic parameter,

Pr-signifies the Prandtl number,

Ec- indicates the Eckert number,

A*- denotes the space temperature-dependent heat source/sink

B*-presents the temperature-dependent heat source/sink".

τ_w −signifies the shear stress

Nu- denotes the Nussult number. These quantities are,

$$k_2 = \frac{\gamma}{bk'}, M^2 = \frac{\sigma B_0^2}{b\rho}, \qquad Pr = \frac{\mu C_p}{K_\infty}, \quad Ec = \frac{b^2 l^2}{Ac_p}$$

$$\tau_w = \frac{\tau^*}{\mu bx\sqrt{b/\gamma}} = f_{\eta\eta}(0), Where\tau^* = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
(12)

$$Nu = \frac{-h}{T_w - T_\infty}T_y = \begin{Bmatrix}\theta_\eta(0)PSTCase \\ 1/\theta(0)PHFCase\end{Bmatrix}$$
(13)

3 Numerical Solution

We utilize the highly efficient shooting technique, as detailed in references De Boor and Conte [21] and Bradshaw and Cebeci [22], in conjunction with a 4th-order Runge-Kutta integration method to address the boundary value issues discussed earlier in the preceding section. In the PST and PHF scenarios, we convert the nonlinear equations (14) and (15) into five first-order ODEs.

$$\frac{df_0}{d\eta} = f_1,$$
$$\frac{df_1}{d\eta} = f_2,$$
$$\frac{df_2}{d\eta} = \frac{(f_1)^2 + M^2 f_1 - f_0 f_2 - 2\beta f_0 f_1 f_2 - k_2 f'}{1 - \beta {f_0}^2},$$
(14)

$$\frac{d\theta_0}{d\eta} = \theta_1,$$
$$\frac{d\theta_1}{d\eta} = Pr[2f_1\theta_0 - \theta_1 f_0] - \mathrm{Ec}\,Pr\,{f_2}^2 + (A * f' + B * \theta).$$

The boundary conditions (9) then take the form,

$$f_0(0) = 0, f_1(0) = 1, f_1(\infty) = 0,$$
$$f_2(0) = 0, \theta_0(0) = 0, \theta_0(\infty) = 0.$$
(15)

Here $f_0 = f(\eta)$ and $\theta_0 = \theta(\eta)$.First finding the missing slopes$f_2(0)$ and $\theta_1(0)$ by shooting technique. The convergence criteria significantly depend on estimations of the firing procedure's initial conditions that are deemed to be fairly reliable. The iterative operation will continue until the relative variation between the most recent iterative values of $f_2(0)$ and the most prior iterative value of $f_2(0)$are equal, with a tolerance of10^{-6}. To get the crucial outcome, integrate the resulting ODE using the 4th-order Runge-Kutta technique after attaining convergence.

4 Results and Discussion

The results of numerical computations for different physical parameters are shown visually in Figures 1 - 11. The 4th order Runge-Kutta with shot technique approach was used to solve the nonlinear ODEs (7 to 9) subject toboundary conditions (5), (9), and (10) numerically. Visual representations and a brief description of the impact of various parameters on the velocity and temperature profiles are provided.

Figures 1 & 2depict a variation of M with $\beta = 1$ on the velocity profile. A rise in the Mcorrespondstoa reduction of the velocity profile along u and v.The applied transverse magnetic field creates Lorentz force drag, which reduces velocity.Figures 3 and 4depict the variation of β with $M = 1$ on the velocity profile. A rise in $\beta$is observed to declinethe velocity profile along u &v. Figs.5 and 6 reveal that, the effect of $\gamma$with $M = \beta = 1$ on the velocity profile. A rise in γ corresponds to a risein velocity profile along u and v.

Fig.7(a) & 7(b) reveal "the effectof Pr on the profiles of temperature for2 special cases PST and PHF. As Princreases the temperature increases monotonously from T_s to T_0. The thermal boundary layer thickness drops significantly For high values of Pr, or low thermal diffusivity.

Figures 8(a) and 8(b) depict the impact of Ec on the temperature profiles for two unique instances, PHF andPST. Ec has the power to raise the fluid's temperature. Higher Ec values, then, suggest a thickening of the thermal boundary layer. Figs. 9(a) and 9(b) show the impact of the space-dependent parameter, A^*, on the temperature profiles for two unique situations, PHF andPST. In the PST and PHF cases, the thermal boundary layer generates energy that causes the temperature to increase in magnitude with raisingvalues of,$A^*(> 0)$ whereas for,$A^*(< 0)$The boundary layer fascinates energy that causes the temperature to significantly decrease with reducing values of$|A^*|$.For some negative values of A^*, it is seen in all these plots that there is a heat transfer from the boundary layer area to the sheet.

Figures 10(a) and 10(b) for the PHF and PST instances illustrate how the temperature-dependent parameter, B^*, affects heat transport. According to these graphs, energy is released for rising values of$B^*(> 0)$which results in temperature increases in PST and PHF situations, but energy is absorbed for falling values of$B^*(< 0)$ which causes the temperature to fall sharply close to the boundary layer.

Figures 11(aand b)shows that, the effect of porosity $\gamma$with $M = \beta = 1$ on the temperature profile. When the porosity parameter israised, the temperature falls in the PST case, while the converse is true in the PHF"case.

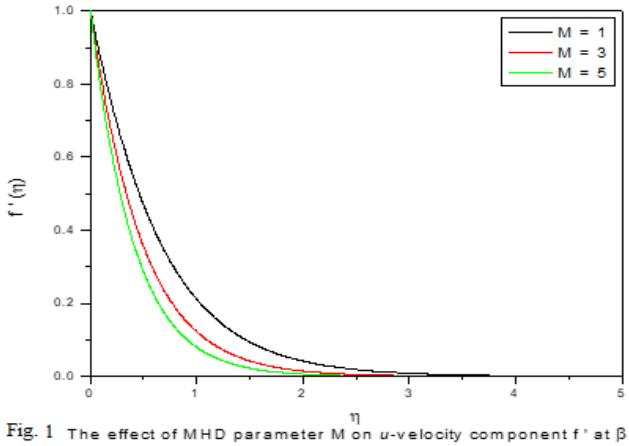


Fig. 1 The effect of MHD parameter M on *u*-velocity component f ' at β =1

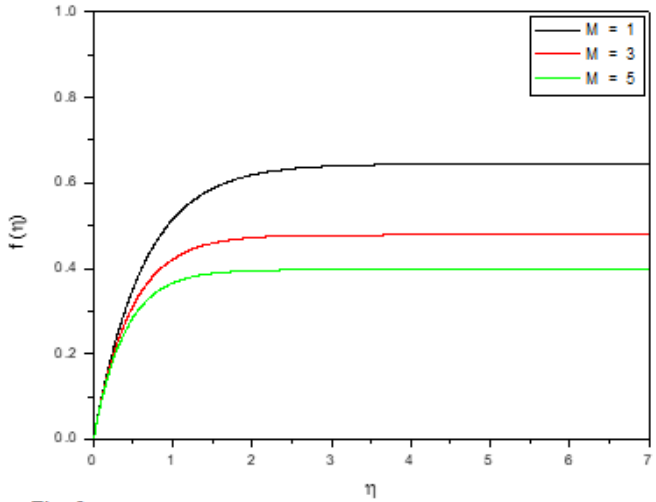


Fig 2.The effect of MHD parameter M on *v*-velocity component f at β =1

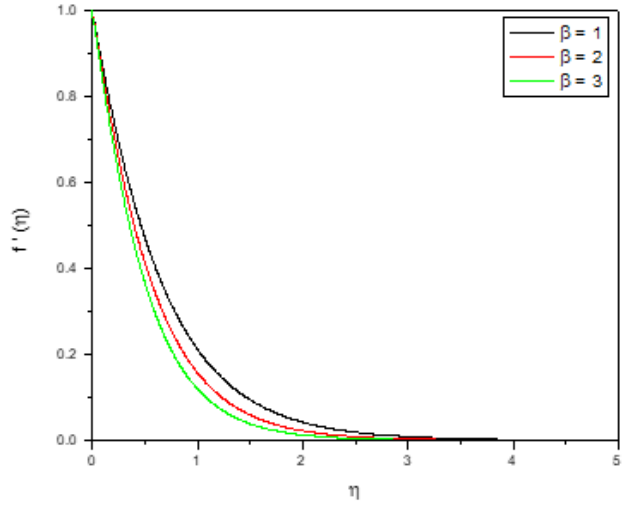


Fig. 3. The effect of elastic parameter β on *u*-velocity component f' at β=1

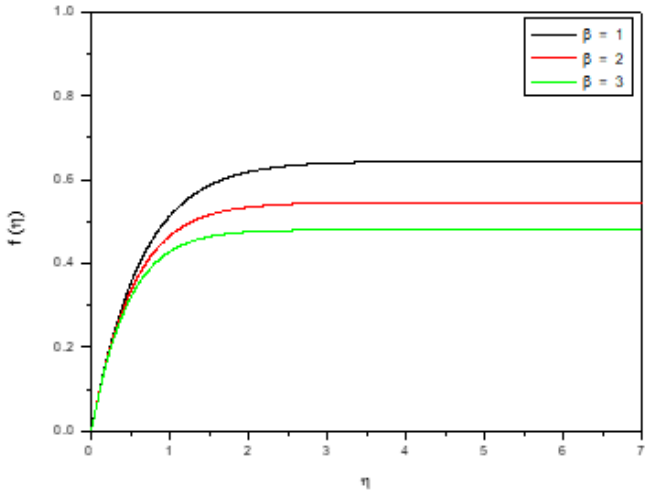


Fig. 4. The effect of elastic parameter β on *v*-velocity component f at β = 1

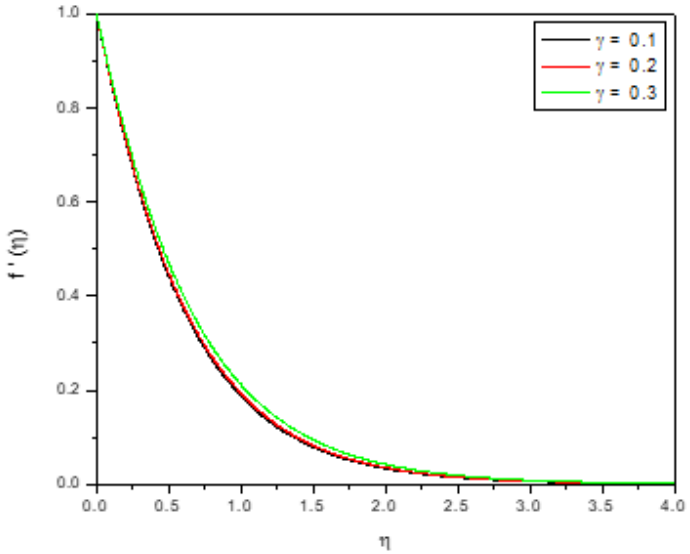


Fig. 5 The effect of Porous parameter γ on *u*-velocity component f at M=β=1

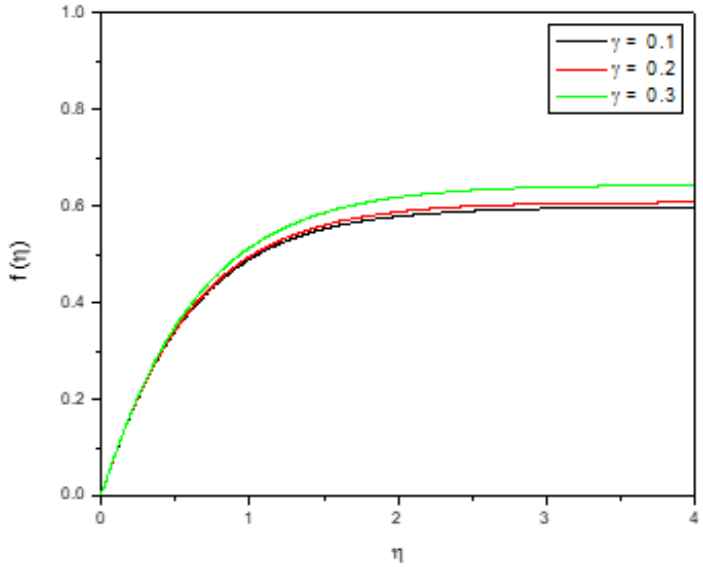


Fig. 6. The effect of Porous parameter γ on *v*-velocity component f at M=β=1

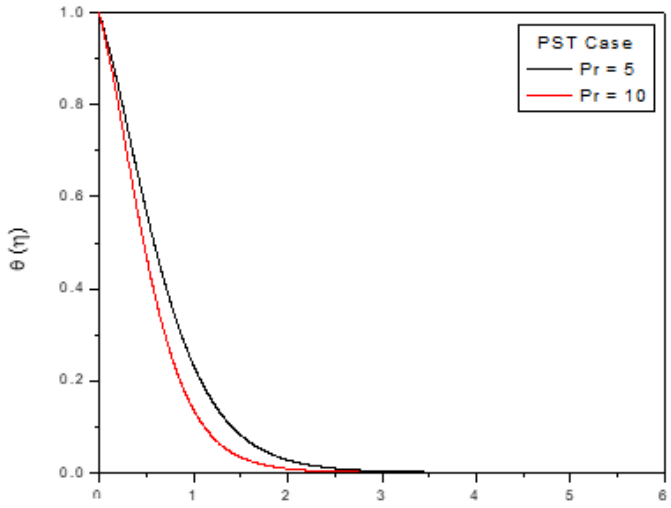


Fig. 7a. The effect of Prandtl number Vs Temperature profile

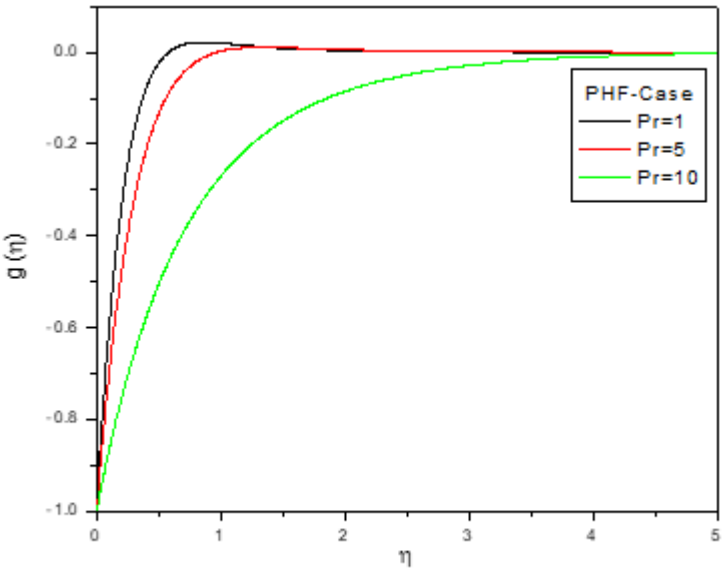


Fig. 7b. The effect of Prandtl number Vs temperature profile

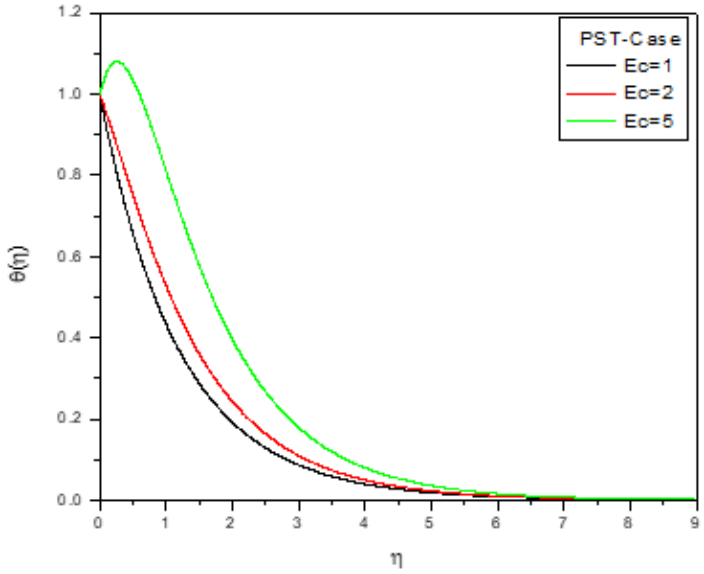


Fig. 8a. The effect of Eckert number Vs temperature profile

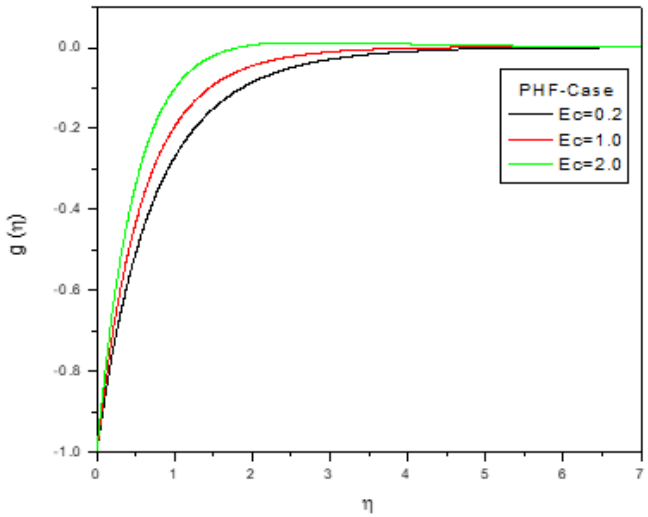


Fig. 8b. The effect of Eckert number Vs Temperature profile

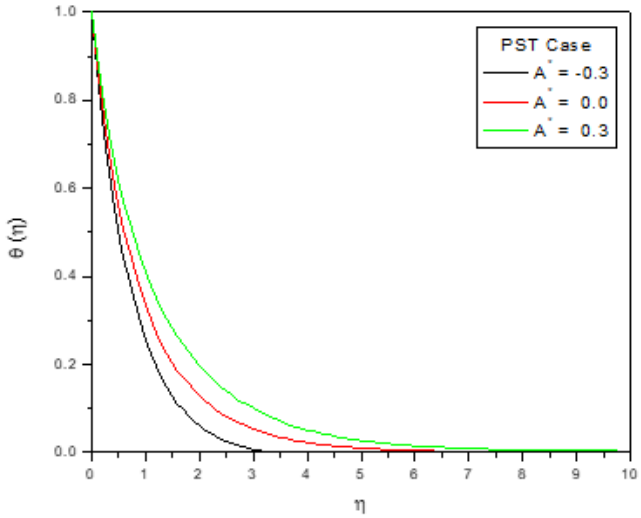


Fig. 9a. The effect of nonuniform heat source Vs Temperature profile

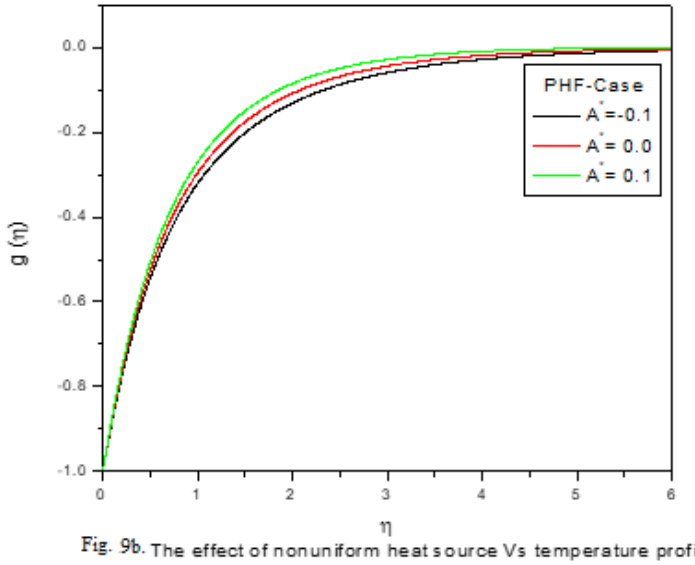


Fig. 9b. The effect of nonuniform heat source Vs temperature profile

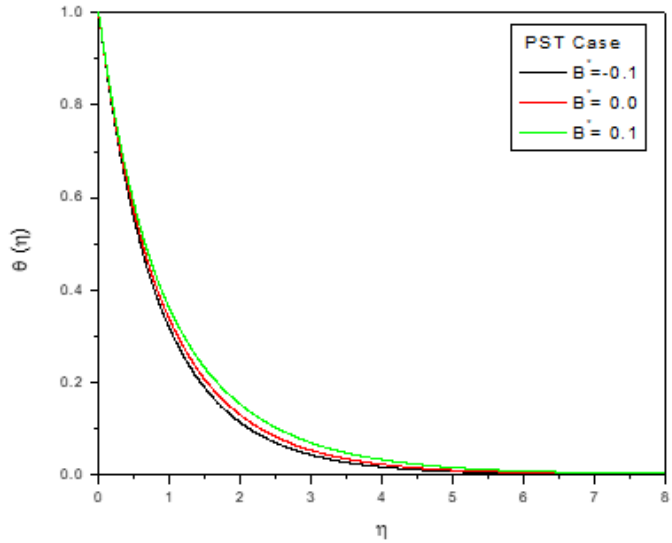


Fig. 10a.The effect of nonuniform heat sink Vs Temperature profile

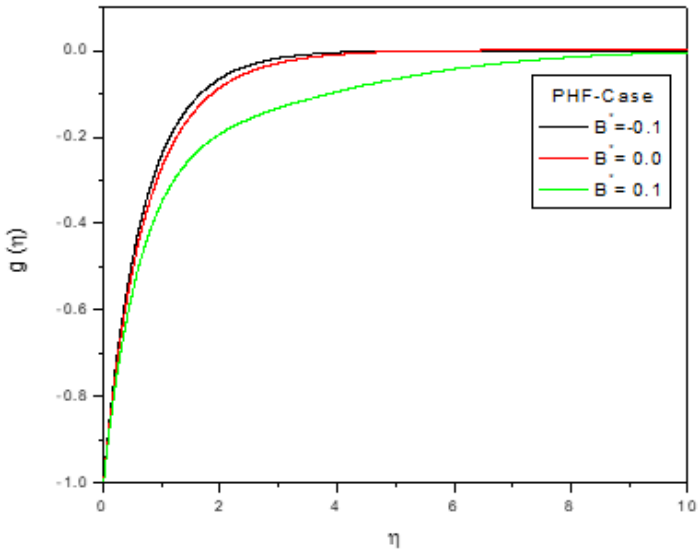


Fig. 10b.The effect of nonuniform heat sink Vs Temperature profile

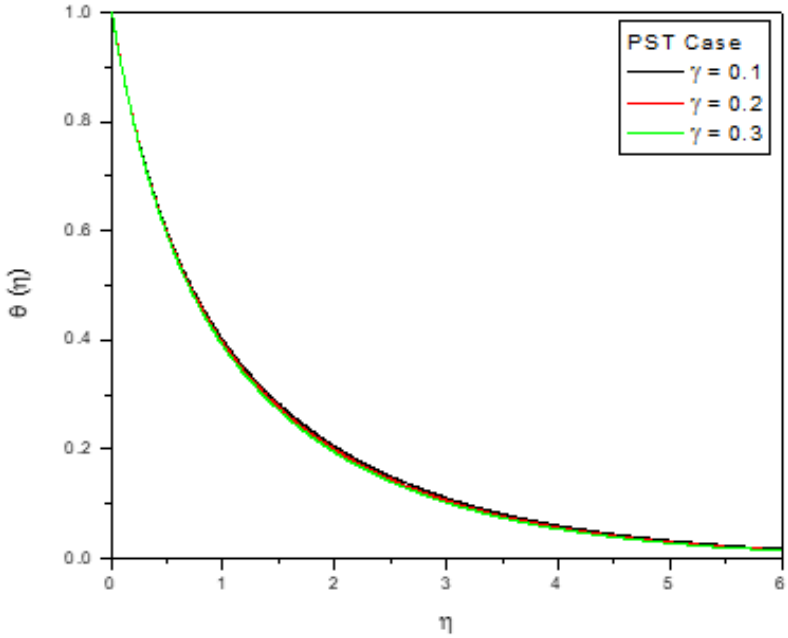


Fig. 11a. Effect of porous parameter γ on temperature profile in PST Case

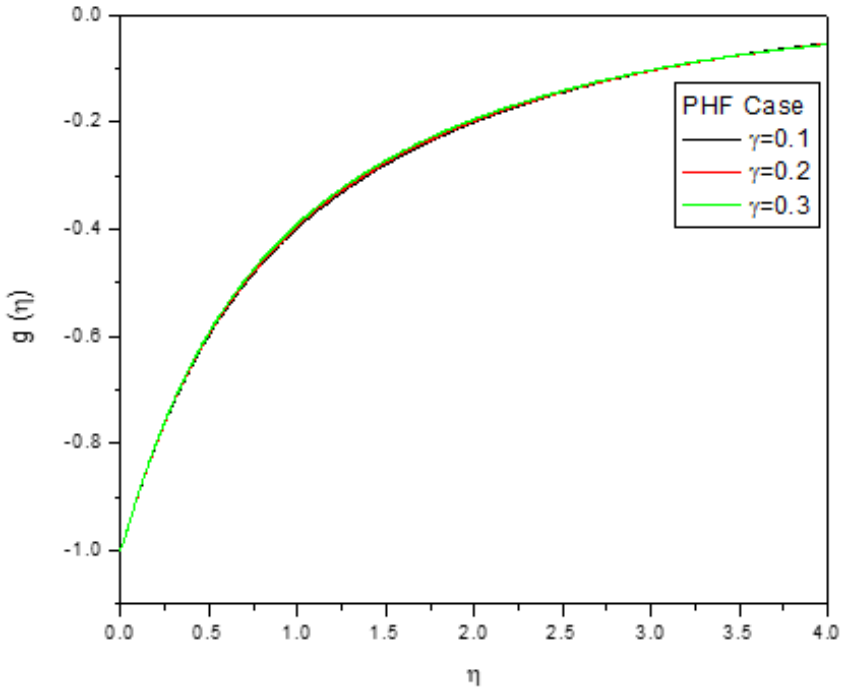


Fig. 11b. Effect of Porous parameter γ on temperature profile in PHF Case

5 Conclusions

1. In a viscous,incompressible,electrically conducting fluid, the existence of a transverse magnetic field causes the velocity field to decrease, leading to a rise in the temperature field.

2. Eckert number measures viscosity. In the incidence of viscous dissipation, dimensionless temperature increases with fluid heating (Ec> 0) and decreases with cooling (E c < 0). Viscous dissipation raises thermal boundary layer temperature.

3. The temperature rises for positive values of the heat source/sink parameters and falls for negative values. Therefore, non-uniform heat sinks are preferable for cooling purposes.

Nomenclature:

c_f	skin friction coefficient
f	dimensionless stream function
h	dimensionless concentration function

k thermal conductivity

M magnetic parameter

Nu_x local Nusselt number

Pr Prandtl number

Re_x local Reynolds number

T temperature of the fluid inside the boundary layer

T_w temperature at the surface of the sheet

T_∞ ambient temperature

U_∞ free stream velocity

u,vvelocity component along x- and y-direction

Greek symbols

η gdimensionless similarity variable

μ dynamic viscosity of the fluid

υ kinematic viscosity of the fluid

ψ stream function

α thermal diffusivity

σ electrical conductivity

Θ dimensionless temperature

∞ condition at the free stream

W condition at the surface

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