

Multiple Constraints Source to K- Terminal Reliability Optimization under Common Cause Failures

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Abstract: Many systems like Gas pipelines, Irrigation systems, Fire Fighting Systems, etc., their satisfactory performance or reliability does not only depend on proper connectivity but also on delivering a required demand. Hence, reliability Optimization (RO) of such networks with Multiple Constraints i.e. Cost, Capacity, and Connectivity (CCC) is a complex decision. The common practices of RO are mostly observed as optimization at the component level compared to other alternative possibilities like right of way, feasibility of structural changes and capital investment for it etc. Till now extensive works have been proposed by many authors on the RO of SKT networks with single or multiple constraints using Exact and Approximate Methods. But RO of SKT Network with multiple constraints under Common Cause Failure (CCF) is a barely studied area. To demonstrate the proposed methodology, a single source to five terminals irrigation network is considered here. A Case Study is presented in three successive sections, Section A- Mathematical Modelling of SKT network applying Exhaustive Search Method to enumerate the successful paths with intended delivery capacity. Section B- Heuristic Approach is used for Reliability Optimization of SKT network with s-identical redundant components under cost constraints. Section C - Demonstrate the effect of CCFs using Poisson Failure processes. In Conclusion Reliability of the SKT network with and without CCF is compared to show the significant effects of CCF. For its analysis, author has considered the irrigation network and assuming the common tendency of network optimization with s-identical components to explain how unknowingly the affect CCF get introduced due to selection of s-identical components especially in an identical operating condition.

Keywords: Conditional Probability, Common Cause Failures, Redundancy Optimization, Source to K-Terminal, Exhaustive Search, s-Identical

1. Introduction

Reliable connectivity with adequate flow capacity is a major concern of today for any supplier to deliver a particular amount at its destination point. In some literature, it has been observed that checking communicability through connectivity between any two vertices or nodes was analyzed for multi-terminal source reliability [1]. However, the quoted protocol may give the solution for reliable connectivity of computer networks. In the last few decades, there is a tremendous development in all kinds of networks which increases the structural, space, and time complexity of reliability computations. So, researchers and scholars are continuously putting their efforts into finding suitable methods and algorithms to reduce the mentioned complexities. To mention a few recent works based on exact approaches are- Corinne Lucet et al have applied the following exact approaches Path Enumeration, Decomposition, and Inclusion-Exclusion Method are

Demonstrated for analyzing the K-terminal reliability to study whether the network will continue to work after the occurrence of some random failures or not. In his study, he stated that using a decomposition-based reduction algorithm greatly helps in linearizing the reliability solutions [2]. Yuchang Mo et al have presented work on K-terminal reliability analysis of a Sensor Cloud System (SCS) which focused on perfect information exchange between every two nodes of a particular network subset K. The proposed methods follow the graph decomposition and rebuilding through communicable vertices. Integrating the proposed method with Binary-Decision-Diagrams (BDD) for real-life large SCS networks, visibly increases the speed of the K-Terminal Reliability Evaluation for real-life large SCS networks [3]. Since Bryant et al have proposed the implementation of the BDD Algorithm for network reliability analysis, it took a good pace in developing the original algorithm to fit into different kinds of networks like s-T, S-K-T, All-Terminal Reliability of directed and non-directed vertices. Zhusheng Pan et al have proposed a method to overcome constraints of accuracy or space efficiency, using the OBDD dependency test for reliability analysis of K-terminal network considering both edge and vertex failures. the proposed modified algorithm shows higher accuracy in network reliability computation with imperfect vertices. Fu-Min Yeh et al have presented BDD as an efficient approach to estimating the reliability of a

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non-directional-terminal network based. The efficacy of this approach is demonstrated by experimenting on multiple large standard networks. A significant result is observed when the reliability of a source-terminal 3 10 all-terminal network could be estimated within 2.4 seconds on a SPARC 20 workstation. It is faster than earlier factoring methods [4,5]. Shihu Xiang and Jun Yang have presented a method to estimate the reliability of an ad-hoc network with K-terminals even in crucial conditions to ensure the operational ability of the networks. The average generalized k-terminal reliability is proposed and based on the Laplace transformation technique and the graph theory [6]. Even if it has many advantages but still it faces certain challenges. These are (a). Generally, identifying the best possible ordering π for a function is NP-hard, even when it is given as an OBDD [7]. (b) No guaranteed polynomial-time algorithm is available for ordering within a constant factor of the optimum unless $P = NP$. Hence, some authors have used the heuristic approach to build comparatively better variable orders for specialized applications like combinational circuits, sequential circuits, a CNF representation, or a set of interacting state machines to derive their Boolean functions respectively [8,9,10,11,12] (c) The other concern is that practically systems may be of repairable in nature. So, as Boolean does not specify the intermediate states, hence cannot be applied in multistate models to derive the functional relations. (d) Another challenge is that widely used BDD packages even now also perform on a sole core of a sole machine, and they hardly can use the available physical memory on high-end machines. Although individual computer systems are developed with enhanced memory, cores, and higher disk capability to align with related issues and, build the ability to work together on a single task by a massive group of machines. (e) But, in most applications of OBDDs, the capability is restricted to deal with bigger and multifaceted problems due to the size of the OBDDs generated. If, it exceeds the machine's memory capacity implementation of the technique may measurably slow well before the threshold because of inadequate memory-system performance (f) Although the newly developed multi core processors help in executing several instruction threads in parallel mode for OBDD operations on single memory space, developing such multi-processor system with rational shared memory is a highly expensive affair. This is really a major challenge to plan OBDD implementations (g) still there are a constant search to have compact Boolean functions, implementation of OBDD for systems having state variables keeping the benefit of key features of OBDD algorithms [13]. But if connectivity with adequate capacity is the measure for reliability optimization, then proposed techniques with simple algorithms may give better and quicker results. Here the complete analysis is

carried out in the following sections as follows- Problem Formulation, SKT Network Reliability Modeling, Reliability Optimization of SKT Network, Effect of CCFs on SKT Network Reliability Optimization, followed by a Case Study and Result Analysis.

1.1 Problem Statements:

- 1) Reliability Analysis based on an independent failure process is studied for following networks like S-T, S-to –All Terminals, S-K-T, etc. based on the application-oriented attribute of different kinds of networks. But the Reliability of SKT terminals under CCFs is still found as a rarely studied area.
- 2) A common practice of any network optimization is observed either by introducing the redundant parallel network or a redundant specific component to the system. The obvious reason for choosing the redundant component/system is to reduce resource expenses and other hassles. But the big mistake is generally made while choosing the option as s-identical elements for redundancy. These practices unknowingly push the system into other failure modes caused by Common Cause Failures (CCFs) especially when operated under the same operating conditions.
- 3) Common Cause Failures defeat the purpose of systems' reliability optimization. If the effect of CCFs is ignored during the analysis, the optimized network shows more optimistic results which is not true. How to incorporate this effect in this analysis due to presence of s-identical components are modeled mathematically for SKT network using Poisson Processes is illustrated here.

1.1 Contribution:

- 1) Reliability of SKT network is generally estimated to assure the service availability of network with desired capability during its run time. Hence, author has considered a system say as non-repairable, because first case of study is as stated minimum K terminals are in operation to serve the purpose out of SNT. Here, to determine the different probable mode of operation or combination of terminals considering binary (1=up, 0=down) status is shown.
- 2) An algorithm is developed to find the probable successful mode of operations and their reliability keeping the capacity constraints as a base for selection of K^{th} terminals using Exhaustive Search Method.
- 3) To compare the results of SKT network reliability due to a) Independent failures only and b) both Independent & Common Cause Failures respectively Authors have optimized the SNT. For this author have considered the common practice of network optimization and optimized it with s-identical

components using Heuristic Approach and re-estimated the Optimized networks' reliabilities as mentioned

2. Methodology

2.1 SKT Network Reliability Modeling:

All hardware is having aging and fatigues which put them in limitation to perform either partially or failed completely over a period of its usage. Ensuring such systems' reliability under all circumstances becomes very crucial, especially when its sub systems are performing with different capacity. Thorough Analysis is required to ensure its reliability by confirming its connectivity with adequate capacity. The aging or fatigue of hardware is improved by either maintaining or replacing that specific component. But in all cases maintenance may not be

feasible due to its manufacturing cartel or other reasons. Moreover, demand for certain systems needs a high level of reliability under any circumstances. In all hardware systems the very common worst possibilities are failure of a single or multiple components. Extensive studies have been observed to address the reliability of SKT network under independent failures only. But as mentioned, very rare studies were found to determine such systems reliability under CCFs. To fill this gap authors have presented here the complete procedure to analyze such NP-hard problems to obtain its exact solution. For a generalized derivation, considering here a system with single Source to N number of Terminals (SNT) with Series Parallel Network Configurations, as Irrigation networks are mostly observed with these configurations only.

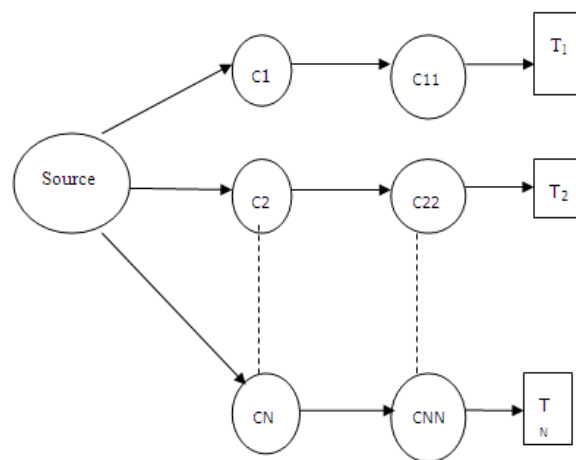


Fig. 1 Sample of SNT Network

Hence, to ensure service reliability of such networks at any point of time, individual terminals bi-states conditions are assumed i.e (for the modes Up=1 or Down=0). These two are EXCLUSIVE -OR situations and are valid for non-repairable system conditions. Also, even if it is repairable but to work under said hypothesis, at least K -terminals with desired capacity must be available at the point of its need. Considering the bi-state conditions, 2^N terminal states will be available with SNT Network structure. Now, Exhaustive Search Algorithm will trace out all those successful states out of 2^N states based on Capacity Constraints (C_{ik}) and thus may be considered the reliability of those states for reliability estimation of SNT network with SKT structure. To apply the Exhaustive Search approach authors have proposed following algorithm in terms of Flow Chart.

Algorithm-

- Determine the combination of K –Terminals using the Exhaustive Search Approach based on the capacity constraint.

- Determine the system reliability expression considering all successful states i.e SKT Terminals out of SNT Network.

The general reliability expression of SKT Network can be mathematically represented as Eqⁿ (1) inclusive of all possible successful states applying Exhaustive Search Approach.

$$R_{skt}(s) = \sum_{i=1}^m \prod_{k=1, k \neq i}^N x_i * p_k * (1 - p_{k+1}) \dots (1)$$

Where, $x_i = \{ 1, \text{ If } C_{ik} \geq y, \text{ output volume or else } = 0$

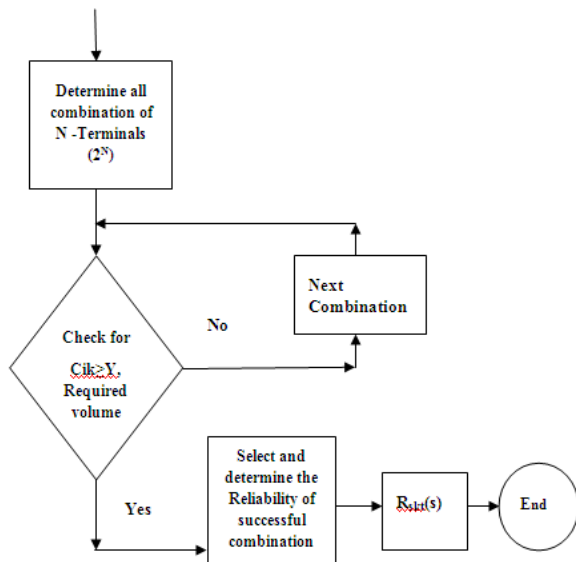


Fig.2. Algorithm for Exhaustive Search Approach

provided,

$$\text{Min}_{\text{cap}} \geq X \text{ ltr/hr for each successful state}$$

where, n = Total no. of states

m = Total no. of successful states

k = Total no. of successful terminals

j = No of unsuccessful terminals out of n

2.1.1 Other Notations

x_i - mode of states

p_i - reliability of unit terminal

q_i - unreliability of unit terminal

$R_{skt}(s)$ -SKT network reliability

$R_{sktoic}(s)$ -Optimized SKT network reliability with s -identical component

$R_{sktoic}(t)$ -SKT network reliability under CCF at “ t ” time instant

$RIC(t)$ -reliability of individual terminal under CCFs

$FIC(t)$ –unreliability of individual terminal under CCFs

s_i -type of subsystems of each terminal

C -Total Cost

C_{xi} -Cost of individual component

2.2 Reliability Optimization of SKT Network

Optimizing the existing systems is a very popular practice of the whole engineering society. General objectives are to minimize the consumed resources and maximize the output of the systems. Also, from the reliability perspective using component or system level redundancy

optimization techniques are very common and preferable selections. Especially, where cost & space occupancy is concerned, component level redundancy optimization is observed as a more preferred option [14,15,16]. For optimization authors have opted to use heuristic method as approximate approaches fails to provide the exact solutions. Heuristic Approach is used here for optimization of the network under the following criteria. This approach helps in achieving the exact optimal solution by iteratively improving the candidate’s value.

2.2.1 Algorithm for Optimization

a) Optimize K-Terminals reliability based on cost constraints:

Heuristic Approach of optimization may be implemented based on historical data or guess work if the system is yet to be configured. Or else optimization may start with the basic configuration of the system assuming all subsystems having only one single component. To prioritize for optimization following two selection measures must be checked before-

- Terminal Selection Measures: As per demand, the terminal with the highest capacity and low reliability is considered on priority.
- Component Selection Measures: Ratio of total reliability due to inclusion of unit component to total cost out of available capital i.e. $\sum R_{xi} / (\sum C_{xi} / C)$.

(b) Optimize the network till all resources are completely utilized.

- Optimized the K-Terminals Reliability based on Cost Constraints.

2.2.2 Stopping Criteria

It is vital to mention that optimization is always opted within the resource limitation. Hence, for optimum utilization of available resources, a stopping limit must be considered. Else, it may mislead the designer for wrong utilization of resources and come up with poor outcomes.

2.3 Effect of Common Cause Failures on SKT Network Optimization

System optimization incorporating s -identical redundant components has been observed as a very common practice in most cases. But the general consequences of s -identical components are that they have a similar tendency to respond when it is operated under the same environmental and physical conditions. Alternatively, due to any implicit or explicit reasons, if one component fails, then there is a high probability to fail multiple identical components.

Hence, optimization of the SKT network with identical redundant components may have a strong tendency to fail the system due to the presence of CCFs, that causes simultaneous failure of multiple s- identical components.

2.3.1 Incorporation of Common CCFs in SKT network reliability analysis

To analyse the effects of Common Cause Failures Probability, equation of Poisons failure processes may be used [17,18, 19] under following assumptions. This defines all dependent failure processes, if a specific component fails within the same group.

2.3.1.1 Assumptions:

- Subsystems are of bi-states with s- identical components.
- Each one of it is exposed to different failure processes. Each failure process encompasses a specific number of components.
- Different failure rates are considered for different failure processes

2.3.1.2 The technique applied to integrate CCF into network reliability analysis under CCFs is briefed as follows-

Algorithm:

- To identify all possible failure processes of respective subsystems.

Reliability of a specified component may fail due to several failure processes. The associated failure processes can be identified from the following Poisson [19,20,21] equation. For example a specific component that is in working condition at time t ,

$$p_m^{(1)}(t) = \exp(-\sum_{r=1}^n r_{n-1}^{-1} C \lambda_r t) \text{ \& \dots\dots (2)}$$

Probability of group of m components all is good for identically distributed components can be estimated as

$$\begin{aligned} p_n^{(m)}(t) &= Pr\{s_1; t\} Pr\{s_2 | s_1; t\} \dots \dots Pr\{s_m | s_1 s_2 \dots \dots s_{m-1}; t\} \\ &= \\ p_n^{(1)}(t) p_{n-1}^{(1)}(t) \dots \dots \dots p_{n-m+1}^{(1)}(t) \\ &= \prod_{k=n-m+1}^n P_k^{(1)}(t) \dots\dots\dots (3) \end{aligned}$$

- Considering CCFs, frame the reliability idiom
- Considering CCFs, Calculate reliability of augmented network.

3. Case Study

The following case study is considered here to implement the complete proposed methodology for comparative analysis of the SKT Network Reliability with and without incorporating the effect of CCFs.





Fig.3. Irrigation Systems.

Section A

3.1 Mathematical Modeling of SKT Network

From the above different projects (Fig.3) designs, it has been observed that irrigation systems network configurations are in a series-parallel structure, where pumps, valves, and connecting pipelines are in a series configuration. To estimate the reliability of such a

network considering connectivity and capacity under cost constraints, authors have considered a single source to five terminals irrigation network is assumed here as shown in Fig.4 below. In this irrigation network, P & V indicates as Pumps and Valves of respective terminals T1, T2, T3, T4, and T5 with subsystems s1 & s2, s3 & s4, s5 & s6, s7 & s8, s9 & s10.

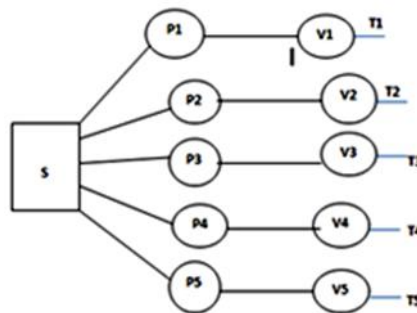


Fig. 4 Five Terminal SKT Network

3.1.1 Assumptions

- Components are of bistate (on /off)
- Every component of subsystems has different failure rates
- Each specific terminal may have different capacity to deliver

- Required minimum total delivery capacity by terminals is 3000 ltr/hr
($0.46\% = 3000/6500$)

Based on stated assumptions, Table 1 indicates component reliability & cost per unit of each subsystem & Table2 represents terminals reliability with mentioned capacity

Table1: Components Reliability & Cost per unit

X_i	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
p_i	0.7	0.75	0.8	0.85	0.9	0.93	0.85	0.87	0.88	0.86
c_i	1	1.5	2	2.4	3	3.3	2.6	2.8	3.3	3

Table2. Terminal Reliability, Cost & Capacity Based on Subsystems Reliability as in Table. 1

	T1	T2	T3	T4	T5
T_i	(s1*s2)	(s3*s4)	(s5*s6)	(s7*s8)	(s9*s10)
p_i	0.525	0.68	0.837	0.7395	0.7568

qi	0.475	0.32	0.163	0.2605	0.2432
ci	2.5	4.4	6.3	5.4	6.3
Cap (in ltr. /hr)	1000	1500	1600	700	1700

Here Table. 2 presents failure rates of various independent & dependent failure processes that may fail due to any independent & dependent implicit reasons.

3.1.2 Process to determine reliability expression of SKT Network under Cost & Capacity Constraints:

Now, the reliability of above SKT Network as considered in Fig. 4 is determined using the modeling approach stated in Eqⁿ (1) considering Minimum Capacity as 3000 ltr/hr is

$$R_{skt}(s) = q_1q_2p_3q_4p_5 + q_1p_2q_3q_4p_5 + q_1p_2p_3q_4q_5 + q_1q_2p_3p_4p_5 + q_1p_2q_3p_4q_5 + q_1p_2p_3p_4q_5 + p_1q_2q_3p_4p_5 + p_1q_2p_3q_4p_5 + p_1q_2p_3p_4q_5 + p_5 + p_1p_2q_3p_4q_5 + p_1p_2p_3q_4q_5 + q_1p_2p_3p_4p_5 + p_1q_2p_3p_4p_5 + p_1p_2q_3p_4p_5 + p_1p_2p_3p_4p_5 = 0.9068$$

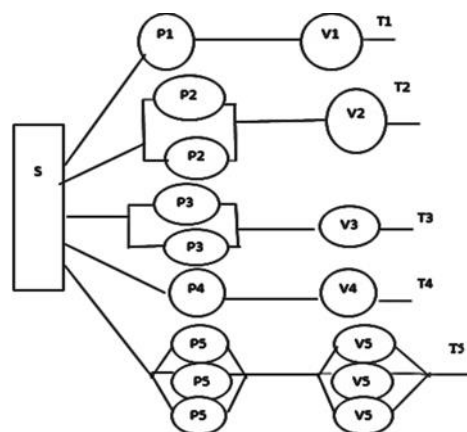


Fig.5 Optimized Network Configuration

The stopping criteria limit is decided here based on value of incremental system reliability with an additional increment of redundant component if goes below 0.01 value.

Section B:

3.2 Optimization of SKT network configuration with s-identical

component

Applying Heuristic Approach as stated above the obtained optimized network configuration is , $xi=\{1,1,2,1,2,1,1,1,3,3\}$, and terminal Configurations $T1=\{1,1\}$, $T2=\{2,1\}$, $T3=\{2,1\}$, $T4=\{1,1\}$, $T5=\{3,3\}$, the new structure of the network is as shown in Fig.5.below and their respective improved reliability a, unreliability, cost and capacity is tabulated in Table.3

Table 3: Optimized Terminal Reliability, Cost & Capacity

Ti	T1 (s1 *s2)	T2 (s3 * s4)	T3 (s5 * s6)	T4 (s7 *s8)	T5 (s9 *s10)
Pi	0.525	0.816	0.9207	0.7395	0.9952
qi	0.475	0.184	0.0793	0.2605	0.0048
Ci	2.5	6.4	9.3	5.4	18.9
Cap (in ltr. /hr)	1000	1500	1600	700	1700

Here, Fig. 6, Fig.7, Fig8, Fig.9 shows the comparative reliability, unreliability cost and capacity of individual terminals respectively based on Table. 3.

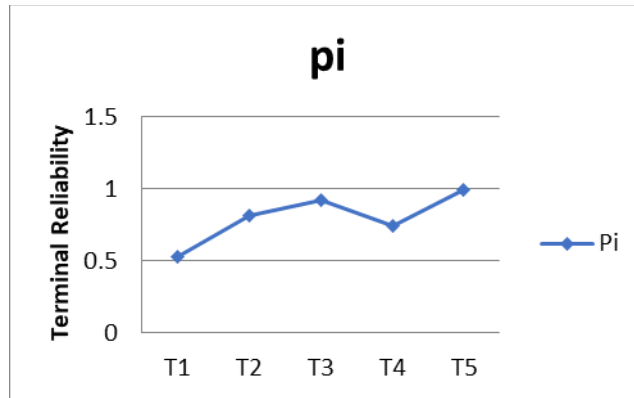


Fig.6. Reliability of Optimized Terminals

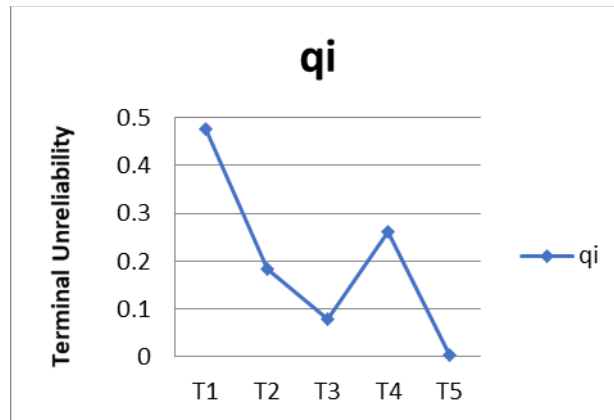


Fig.7. Unreliability of Optimized Terminals

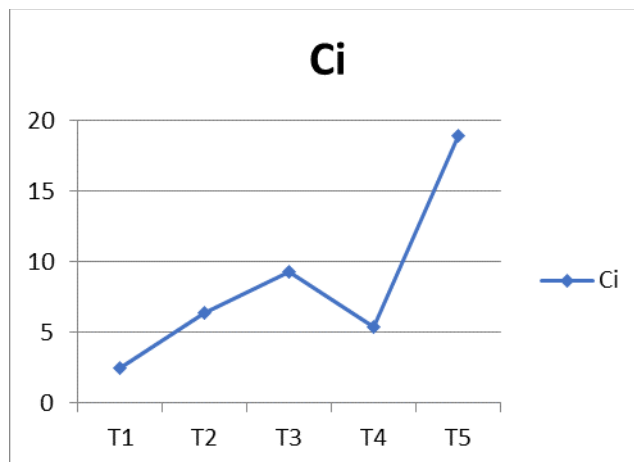


Fig.8. Cost of Optimized Terminals

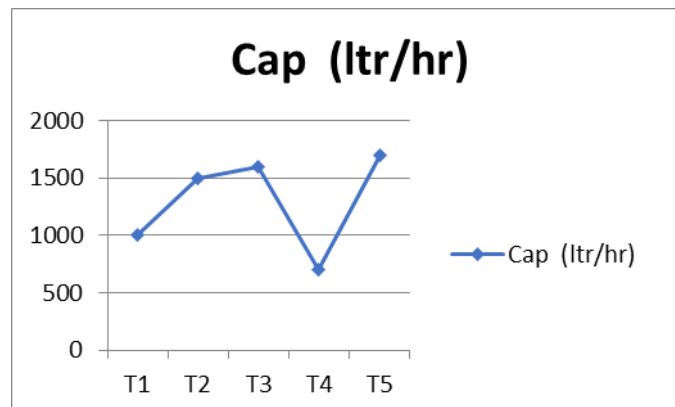


Fig.9. Capacity of Optimized Terminals

3.2.1 Reliability of optimized SKT network is calculated by replacing optimized terminal reliability as in Table 3.

$$R_{skt}(s) = q_1q_2p_3q_4p_5 + q_1p_2q_3q_4p_5 + q_1p_2p_3q_4q_5 + q_1q_2p_3p_4p_5 + q_1p_2q_3p_4q_5 + q_1p_2p_3q_4q_5 + p_1q_2q_3q_4p_5 + p_1q_2p_3q_4p_5 + p_1q_2p_3p_4q_5 + p_1p_2q_3q_4p_5 + p_1p_2q_3p_4q_5 + p_1p_2p_3q_4q_5 + q_1p_2p_3p_4p_5 + p_1q_2p_3p_4p_5 + p_1p_2p_3p_4p_5 = 0.9904$$

Section C:

3.3 Effect of Common Cause Failures on SKT Network Reliability Optimization

As stated above, based on the concept of Poisson Failure Process determination techniques, CCFs and independent failure processes for subsystems with s- identical components are re-estimated here as follows-

Expanding the poisons equations (2) & (3), subsystem with 2- parallel s-identical components and respective reliability considering both independent and CCF estimated as

$$R_{IC}(t) = 2\rho_2^{(1)}(t) - \rho_2^{(2)}(t)$$

3. RESULT NALYSIS AND CONCLUSION:

Table 4: Comparative Reliability Results of SKT Network

Reliability of SKT Network due to independent failure	Reliability of Optimized SKT Network due to independent failure	Reliability of Optimized SKT due to independent failure and CCFs
.9068	0.9904	0.9496

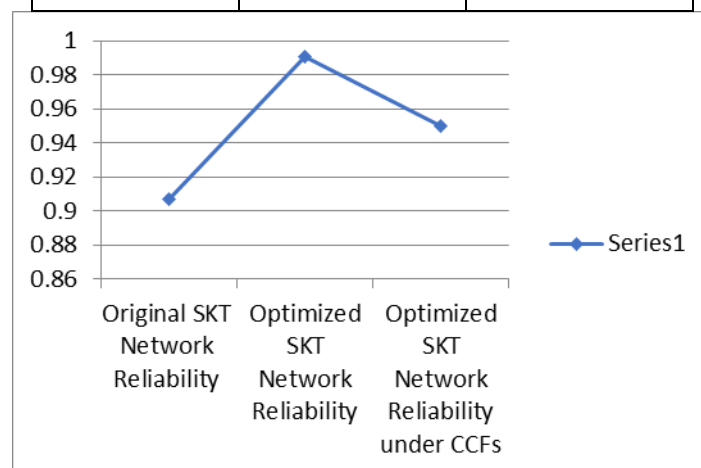


Fig.8. Reliability of SKT Network

From the above computational & graphical analysis of multiple constraints SKT network, it has been observed that if the network is augmented with s-identical redundant components the system becomes more prone to be affected by CCFs. And it overthrows the objective of

$$= 2\rho_2^{(1)}(t) - \rho_2^{(1)}\rho_1^{(1)}(t)$$

Similarly, for the subsystem with 3- Parallel s-identical components, reliability under CCF can be estimated as

$$R_{IC}(t) = 3\rho_3^{(1)}(t) - 3\rho_3^{(2)}(t) + 3\rho_3^{(3)}(t)$$

$$= 3\rho_3^{(1)}(t) - 3\rho_3^{(2)}\rho_2^{(2)}(t) + \rho_3^{(1)}\rho_2^{(1)}\rho_1^{(1)}(t)$$

Like ways reliability of subsystems may be calculated for any number of components if redundant elements are present. Table.3. presents new failure rates of Terminals that may fail due to any independent & dependent implicit reasons.

Integrating CCFs effects, reliability idiom of above augmented network (Fig. 2) is

$$R_{skt}(s) = F_{11c}F_{21c}R_{31c}F_{41c}R_{51c} + F_{11c}R_{21c}F_{31c}F_{41c}R_{51c} + F_{11c}R_{21c}R_{31c}F_{41c}F_{51c} + F_{11c}F_{21c}R_{31c}R_{41c}R_{51c} + F_{11c}R_{21c}F_{31c}R_{41c}R_{51c} + F_{11c}R_{21c}R_{31c}F_{41c}F_{51c} + F_{11c}R_{21c}R_{31c}R_{41c}F_{51c} + R_{11c}F_{21c}F_{31c}R_{41c}R_{51c} + R_{11c}F_{21c}R_{31c}F_{41c}R_{51c} + R_{11c}F_{21c}R_{31c}R_{41c}F_{51c} + R_{11c}F_{21c}R_{31c}R_{41c}R_{51c} + R_{11c}R_{21c}F_{31c}F_{41c}R_{51c} + R_{11c}R_{21c}F_{31c}R_{41c}F_{51c} + R_{11c}R_{21c}F_{31c}R_{41c}R_{51c} + R_{11c}R_{21c}R_{31c}F_{41c}F_{51c} + R_{11c}R_{21c}R_{31c}F_{41c}R_{51c} + R_{11c}R_{21c}R_{31c}R_{41c}F_{51c} + R_{11c}R_{21c}R_{31c}R_{41c}R_{51c}$$

different design of the component, different materials used, different make, or different technology used for operating the component may help in avoiding the system from strong CCF effects.

Data Availability Statements:

Here authors declare that data supporting the findings of this study are available within the articles.

Compliance with Ethical Standards:

Disclosure of potential conflicts of interest: Authors declare that the above analysis is not supported by any funding agencies and do not have any conflict of interest in any financial or nonfinancial Matters.

Research involving Human Participants and/or Animals: It does not include any experimental or non-experimental analysis of humans or animals.

Informed consent: It does not include any experimental or non-experimental analysis of humans or animals. Hence, it is not applicable.

Declarations

Conflict of interest Authors declare that they have no conflict of interest

Consent to Participate All authors have the consent to participate.

Consent to Publish All authors have consented to publish the paper in this journal.

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