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Optimal Capacity and Placement of Distributed Generation using Sequential Quadratic Programming

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Abstract: The growing need for electrical energy is making distributed generation (DG) more significant. Reducing distribution system loss is one of the main goals of distributed generation planning. The ideal placement and scale of distributed generation (DG) are crucial for minimizing losses. This work uses an optimization problem to study the optimal DG placement and sizing, with the objective as reduction of active power losses. These issues are expressed using the Sequential Quadratic Programming (SQP) approach as constrained linear optimization problems. In order to solve the optimal position and sizing problem of distributed generators, the proposed method is extensively proven on IEEE-15 bus and IEEE-33 bus radial distribution systems. Simulation results using the DG demonstrate satisfactory improvements in terms of power loss reduction and voltage profile enhancement.

Keywords: *Optimal Location, Distributed Generation (DG), Sequential Quadratic Programming method (SQP), Voltage profile enhancement.*

1. Introduction

The electric power sector is one of the world's biggest consumer markets. For example, in the United States, purchases of electric energy account for 3% of GDP and are growing at a faster rate than the country's economic growth. An estimated 50% of the cost of energy goes toward fuel, 20% goes into generation, 5% goes toward transmission, and 25% goes toward distribution [1]. Every customer's service entrance must receive energy from distribution networks at the proper voltage rating. In comparison to transmission levels, distribution levels have a lower X/R ratio, which results in more losses and also reduction in voltage magnitude. Literature [2] has shown that at the distribution level, real power losses account for about 13% of the total power produced. The financial problems and general effectiveness of distribution utilities are directly impacted by such non-negligible losses. Distribution power losses are typically reduced by using various voltage control devices to properly dispatch reactive power control devices [3].

Distributed generation (DG) units generally have advantages like decrease in system losses, improvement in voltage profile. However, DG deployment should be optimally done to appropriately use these advantages. This study aims to handle the optimal distribution network DG location and sizing challenge. The optimizations tasks are resolved by applying the deterministic Sequential Quadratic Programming (SQP) technique.

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The aim of the single-objective optimization problem is to minimize the total real power losses in order to determine the best location and amount of distributed generation. In this paper, the effects of integrating single and multiple DGs are also examined. To validate the suggested techniques, two distribution test network topologies - radial and meshed - are chosen, and the outcomes are shown.

2. Problem Formulation

Minimization of the total power losses in the system forms the single-objective function.

$$Minimize\ P_{Loss}(X) \tag{1}$$

Where the complete active power loss will be represented using the following equation:

$$P_{LOSS} = \sum_{k=1}^{NS} G_k \left(|V_i|^2 + |V_j|^2 - 2|V_i||V_j|\cos(\delta_i - \delta_j) \right) \tag{2}$$

Based in nonlinear load flow equations, power balance is determined by subtracting power flows taken out of a bus and adding up all of the complicated power flows that are injected into each bus in the distribution system. This forms the equality constraint of the optimization.

$$P_{DGi} - P_{Di} - \sum_{j=1}^{NB} |V_i||V_j||Y_{ij}|\cos(\delta_i - \delta_j - \varphi_{ij}) = 0 \tag{3}$$

$$Q_{DGi} - Q_{Di} - \sum_{j=1}^{NB} |V_i||V_j||Y_{ij}|\sin(\delta_i - \delta_j - \varphi_{ij}) = 0 \tag{4}$$

where,

P_{DGi}, Q_{DGi} - active and reactive power delivered by DG at bus i
P_{Di}, Q_{Di} - active and reactive power demand at bus i
$|Y_{ij}|$- the magnitude of the ij^{th} element of the admittance matrix
ϕ_{ij} - the angle of the ij^{th} element of the admittance matrix
NB - the total number of buses

The limits on the line flows are expressed using their thermal limits. This forms one of the inequality constraints.

$$S_{ij} \le S_{ij}^{\max} \quad (5)$$

$$S_{ji} \le S_{ji}^{\max} \quad (6)$$

where,

$S_{ji}^{\max}$ - apparent power which is maximum allowable for branch i - j
S_{ij} - apparent power flow that has to be transmitted from bus i to bus j

The output power of DG has minimum and maximum bounds and the power should be less than substation power, and it forms another inequality constraint.

$$\sum_{i=1}^{nDG} (P_{DGi} + jQ_{DGi}) \le P_{ss} + jQ_{ss} \quad (7)$$

$$P_{DGi}^{\min} \le P_{DGi} \le P_{DGi}^{\max} \quad (8)$$

The magnitudes and angles of bus voltages also need to be between defined limits, this is another inequality constraint.

$$\left|V_i^{\min}\right| \le \left|V_i\right| \le \left|V_i^{\max}\right| \quad (9)$$

$$\left|\delta_i^{\min}\right| \le \left|\delta_i\right| \le \left|\delta_i^{\max}\right| \quad (10)$$

3. Sequential Quadratic Programming

The nature of optimization objective and its constraints are nonlinear in nature. The Sequential Quadratic Programming (SQP) method [4-6], is chosen here to solve the DG optimization problem, owing to its correctness, efficiency, and high percentage of successfully solved test problems.

The basic idea behind SQP is to use Taylor's expansion to create linear models of the constraints and a quadratic model of the objective function in order to design the optimizing expressions at the present point, x(k). These are then resolved at every iteration in order to identify a fresh search direction (d) and an improved solution x(k+1). This approach to unconstrained reduction is quite similar to Newton's method [7]. When we solve the general optimization problem using Taylor's expansion, we obtain:

$$f(x) \approx f(x^k) + \nabla f(x^k)^T (x - x^k) + \frac{1}{2}(x - x^k)^T \nabla^2 (x^k)(x - x^k) \quad (11)$$

$$h(x) \approx h(x^k) + \nabla h(x^k)^T (x - x^k) \quad (12)$$

$$g(x) \approx g(x^k) + \nabla g(x^k)^T (x - x^k) \quad (13)$$

Thus, the QP sub problem will have the form:

Minimize: $f(x^k) + \nabla f(x^k)^T d + \frac{1}{2} d^T H^k d$ (14)

$$Subject\ to: h(x^k) + \nabla h(x^k)^T d = 0 \quad (15)$$

$$g(x^k) + \nabla g(x^k)^T d \le 0 \quad (16)$$

In order to apply the Lagrangian multipliers method, the SQP first converts the constrained optimization problem into a Lagrangian function. Next, iteratively solves the unknown variables using the Quasi-Newton method, while also satisfying conditions known as the Karush-Khun-Tucker (KKT) conditions. The expression for this Lagrangian function is given as:

$$L(x, \lambda, \mu) = f(x) + \lambda^T h(x) + \mu^T g(x) \quad (17)$$

where,

λ - Equality Lagrange multiplier
μ - Inequality Lagrange multiplier

The formulation of the QP sub-problem is:

Minimize:

$$f(x^k) + \nabla f(x^k)^T d + \frac{1}{2} d^T \nabla^2 L(x, \lambda, \mu) d \quad (18)$$

$$Subject\ to: h(x^k) + \nabla h(x^k)^T d = 0 \quad (19)$$

$$g(x^k) + \nabla g(x^k)^T d \le 0 \quad (20)$$

where,

$\nabla^2 L(x, \lambda, \mu)$ is the Hessian of the Lagrange function

This results in the SQP method becoming locally converged after applying Newton's method:

$$\begin{pmatrix} \nabla L(x_k, \lambda_k, \mu_k) \\ h(x_k) \\ g_A(x_k) \end{pmatrix} = 0 \quad (21)$$

By resolving the Quasi-Newton, the QP sub-problem solution can be found as follows:

$$\nabla^2 L(x_k, \lambda_k, \mu_k) \begin{pmatrix} d \\ \upsilon_\lambda \\ \upsilon_\mu \end{pmatrix} = \nabla L(x_k, \lambda_k, \mu_k) \quad (22)$$

$$\begin{pmatrix} \nabla^2 L(x_k, \lambda_k, \mu_k) & \nabla h(x_k) & \nabla g(x_k) \\ \nabla h(x_k)^T & 0 & 0 \\ \nabla g(x_k)^T & 0 & 0 \end{pmatrix} \begin{pmatrix} d \\ \upsilon_\lambda \\ \upsilon_\mu \end{pmatrix} = \begin{pmatrix} \nabla f(x_k) + \lambda^T \nabla h(x) + \mu^T \nabla g(x) \\ h(x_k) \\ g(x_k) \end{pmatrix}$$

(23)

For the k[th] iteration,

$$\begin{pmatrix} x_{k+1} \\ \lambda_{k+1} \\ \mu_{k+1} \end{pmatrix} = \begin{pmatrix} x_k \\ \lambda_k \\ \mu_k \end{pmatrix} + \begin{pmatrix} d \\ \upsilon_\lambda \\ \upsilon_\mu \end{pmatrix} \quad (24)$$

Rearranging,

$$\begin{pmatrix} \nabla^2 L(x_k, \lambda_k, \mu_k) & \nabla h(x_k) & \nabla g(x_k) \\ \nabla h(x_k)^T & 0 & 0 \\ \nabla g(x_k)^T & 0 & 0 \end{pmatrix} \begin{pmatrix} d \\ \lambda_{k+1} \\ \mu_{k+1} \end{pmatrix} = \begin{pmatrix} \nabla f(x_k) \\ h(x_k) \\ g(x_k) \end{pmatrix} \quad (25)$$

Each iteration of the QP subproblem requires the calculation of the Lagrangian function's Hessian. Rather than computing the Hessian matrix (B), the Quasi-Newton approach estimates it. The BFGS update formula (Broyden, Fletcher, Goldfarb, and Shanno formula) is considered here for the solution as follows.

$$r_k = \theta_k y_k + (1-\theta_k) B_k s_k \quad (26)$$

$$s_k = x_{k+1} - x_k \quad (27)$$

$$y_k = \nabla L(x_{k+1}, \lambda_{k+1}, \mu_{k+1}) - \nabla L(x_k, \lambda_k, \mu_k) \quad (28)$$

$$\theta_k = \begin{cases} 1 & if \;\; s_k^T y_k \geq 0.2 s_k^T B_k s_k \\ \dfrac{0.8 s_k^T B_k s_k}{s_k^T B_k s_k - s_k^T y_k} & f \;\; s_k^T y_k < 0.2 s_k^T B_k s_k \end{cases} \quad (29)$$

Then we can update B_{k+1} using,

$$B_{k+1} = B_k - \frac{B_k s_k s_k^T B_k}{s_k^T B_k s_k} + \frac{r_k r_k^T}{s_k^T r_k} \quad (30)$$

4. Simulation Results Of Ieee-15 & Ieee-33 Bus Systems

A. Radial Distribution System (IEEE-15 BUS)

An IEEE-15 bus radial distribution feeder operating at a voltage level of 12.66 KV, consisting of 14 branches has been considered as the first test system. It has 3802 kW of active and 2694 kVAR of reactive loads in the system. The following figure represents the single line diagram of the IEEE-15 bus radial distribution system.

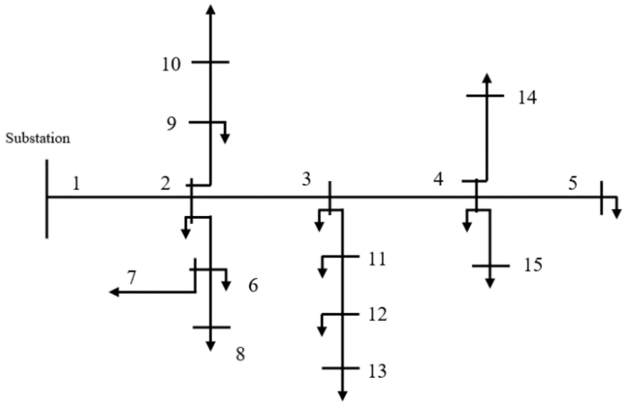

Fig 1: 15-bus radial distribution system single-line diagram

Case I: Installing One DG

By placing a DG at each candidate bus in a 15-bus radial distribution system, the suggested approach was implemented. The ideal size of the DG and the related real power losses are displayed in Table 1, and the voltage magnitude at each system bus is displayed in Table 2. The active power losses have decreased by approximately 2.3%, from 376.3 kW to 289.1 kW, after the installation of the **467 KVA DG** at **bus 13**. The voltage profile improved after the placement of D and the same is represented in Figure 2.

Table 1: Real and Reactive power losses without and with single DG

Branch No.	**P Loss without DG** (kW)	**P Loss with DG** (kW)	**Q Loss without DG** (kVAR)	**Q Loss with DG** (kVAR)
2	235.1	182.4	230.0	178.4
3	70.2	44.9	68.7	43.9
4	15.2	6.2	14.9	6.1
5	0.3	0.3	0.2	0.2
6	0.9	0.9	0.6	0.6
7	0.2	0.2	0.1	0.1
8	22.0	21.9	14.8	14.8
9	5.9	5.9	4.0	4.0
10	2.9	2.9	2.0	2.0
11	13.5	13.4	9.1	9.0
12	3.7	3.7	2.5	2.5
13	0.5	0.5	0.3	0.3
14	1.3	1.4	0.9	1.0
15	4.5	4.5	1.8	1.8
Total	**376.3 kW**	**289.1 kW**	**349.9 kVAR**	**264.7 kVAR**

Table 2: Voltages before and after the DG Placement

Bus No.	**Voltages without DG (pu)**	**Voltages with DG (pu)**	**% Voltage Improvement**
1	1	1	0.00

Bus No.	Voltages without DG (pu)	Voltages with DG (pu)	% Voltage Improvement
2	0.9815	0.9838	0.23
3	0.9721	0.9764	0.44
4	0.9684	0.9742	0.60
5	0.9677	0.9735	0.60
6	0.9745	0.9768	0.24
7	0.9721	0.9744	0.24
8	0.9727	0.9750	0.24
9	0.9803	0.9826	0.23
10	0.9797	0.9820	0.23
11	0.9675	0.9718	0.44
12	0.9647	0.9690	0.45
13	0.9638	0.9681	0.45
14	0.9668	0.9759	0.94
15	0.9659	0.9716	0.59

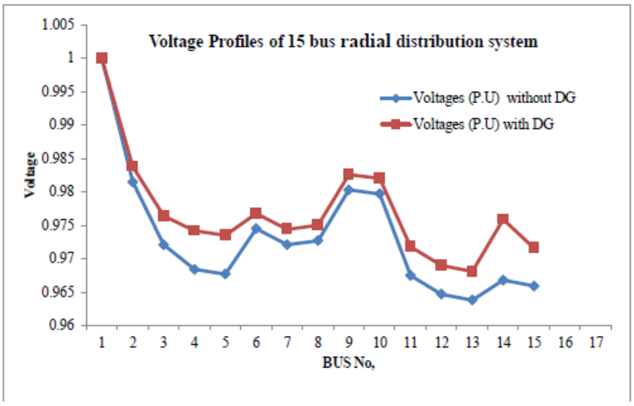

Fig 2: Comparison of Voltage magnitudes with and without DG for IEEE-15 bus system

Case II: Installing Two DGs

Two DGs were installed in order to implement the suggested method. The DG optimum size and associated real power losses are displayed in Table 3. SQP determines that the best place for DG is between **buses 4 and 6** with capacities of **760.1 kW** and **466.4 kW**. The apparent power losses at these buses decreased by approximately 5.9% after the DG was installed. Table 4 illustrates how the voltages have improved with the integration of DG at Bus Numbers 4 and 6. The results obtained are verified, and voltage profiles at buses 4 and 6 are also improved to 1.84 % and 1.12 %, respectively.

Table 3: Real and Reactive power without and with DGs

	P Loss without DG (kW)	P Loss with DG (kW)	Q Loss without DG (kVAR)	Q Loss with DG (kVAR)
2	235.1	90.2	230.0	87.2
3	70.2	9.5	68.7	9.2
4	15.2	1.8	14.9	1.7
5	0.3	0.3	0.2	0.3
6	0.9	0.9	0.6	0.9
7	0.2	0.2	0.1	0.2
8	22.0	25.8	14.8	22.8
9	5.9	8.8	4.0	7.9
10	2.9	3.2	2.0	2.2
11	13.5	0.3	9.1	0.3
12	3.7	3.7	2.5	3.2
13	0.5	0.5	0.3	0.4
14	1.3	1.6	0.9	1.3
15	4.5	2.8	1.8	2.2
TOTAL	**376.3 kW**	**149.7 kW**	**349.9 kVAR**	**139.9 kVAR**

Table 4: Voltages (P.U) before and after placement of DG

BUS No.	Voltages (P.U) without DG	Voltages (P.U) with DG	% Voltage Improvement
1	1	1	0.00
2	0.9815	0.9852	0.38
3	0.9721	0.9821	1.03
4	0.9684	0.9862	1.84
5	0.9677	0.9812	1.40
6	0.9745	0.9854	1.12
7	0.9721	0.9833	1.15
8	0.9727	0.9833	1.09
9	0.9803	0.9869	0.67
10	0.9797	0.9847	0.51
11	0.9675	0.9812	1.42
12	0.9647	0.9852	2.13
13	0.9638	0.9864	2.34
14	0.9668	0.9884	2.23
15	0.9659	0.9883	2.32

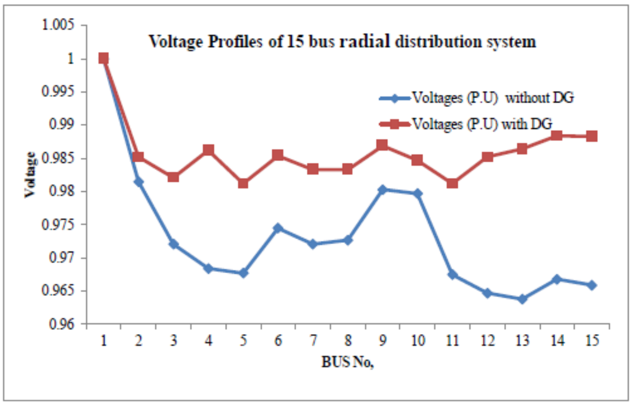

Fig 3: Voltages of IEEE-15 bus system after placing two DGs

B. Meshed Distribution System (33-BUS)

A meshed distribution system with 33 buses and 37 branches, operating at a voltage level of 12.66 kV is considered as the second case study. This system has 2300 kVAR and 3715 kW of reactive and active loads, respectively. Figure 4 displays the single line of the meshed distribution system.

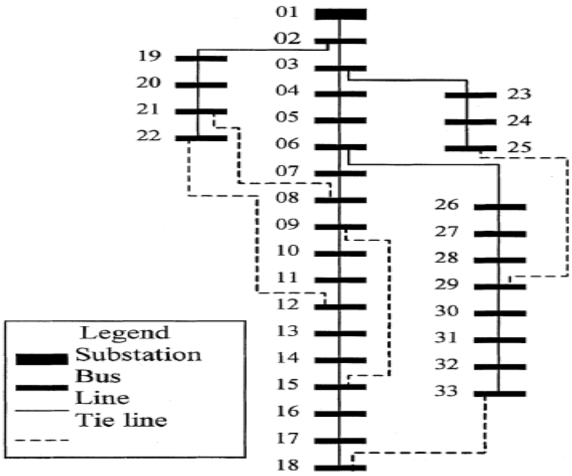

Fig 4: 33-bus meshed distribution system single-line diagram

Case I: Installing One DG

The ideal DG sizing issue for installing a single DG was resolved for each of the 33 busses. The cumulative active power loss is decreased from 2.02 MW, without DG, to 1.18 MW by placing the DG at bus **23** with a capacity of **4370 kVA**, with an approximate reduction of 6.5% in active power losses. Voltage profiles are also improved across all the buses, as Figure 5 illustrates.

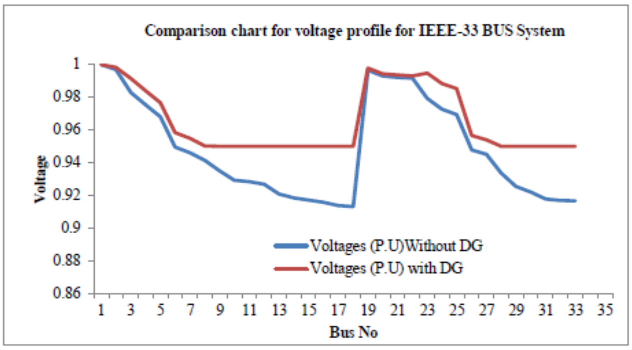

Fig 5: The 33-Bus radial distribution system's voltage profile comparison

Case 2: Installing two DGs

For the installation of two DGs, the ideal DG site and sizing challenge was resolved. The outcomes are displayed in Table 4.9, which also displays the matching total real and reactive power losses for each system bus when installing an ideal DG size. The best place for DG, as assessed by SQP, is between buses 14 and 30. With the two DGs placed at **busses 14 and 30**, respectively, with a power output of **508 kW** and **838 kW**. Additionally, bus 14 and bus 30 have improved voltage profiles to 8.18% and 8.44%, respectively.

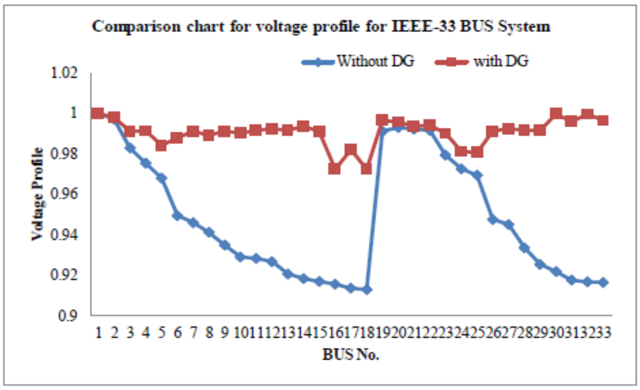

Fig 6: Voltage profile of 33-Bus radial distribution system

5. Conclusion

With the ability to minimize costs, reduce power losses, improve voltage profiles, and reduce complexity, interdependencies, and inefficiencies associated with onsite power generation, transmission, and distribution networks, distributed generation (DG) systems are the ideal solution for today's and tomorrow's power generation and distribution systems. These systems can meet the demanding needs of consumers in an environmentally responsible and cost-effective manner.

This paper looked into the best locations and sizes for distributed generation (DG) within distribution networks. The goal of the single-objective optimization issue was to use total real power losses to estimate the ideal location and size of a distributed generator (DG). Sequential quadratic programming (SQP) is used to minimize this objective. The two case studies having varying configurations, 15-bus radial and 33-bus meshed systems are used to study single and two DG installation instances. A case without DG was contrasted with the outcomes. It was demonstrated that selecting the right DG location and size significantly affects reducing power losses and enhancing voltage profiles.

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