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Interval Valued Q- Fuzzy Vague Subhemirings Of A Hemiring

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Abstract: The output of this paper is to initiate the idea of a combination of fuzzy vague parameter set also we stated a number of combinations of Q - fuzzy vague parameter set properties and applications of a combination of Q- fuzzy vague parameter sets in taking in decision making problems and give the numerical example.

Keywords: Vague sets, Fuzzy vague sets. Q-fuzzy sets, Q-fuzzy vague sets, Combination $Q-$ fuzzy vague parameter sets, Decision taking.

1. Introduction

Arithmetical structures assume a conspicuous job in science with applications in such subjects as hypothetical material science, physics computer Science, coding hypothesis, and topological spaces. Specialists ought to keep on exploring dynamic arithmetical ideas and results in the more extensive structure of the fuzzy setting. One structure that numerical applications are most widely examined is the cross-section hypothesis. In actuality, issues in financial aspects, designing, sociologies, and what's more, clinical science don't generally include fresh information. We much of the time can't utilize conventional numerical strategies due to different uncertainties introduced in these applied issues. To defeat these uncertainties, speculations like the fuzzy set, intuitionistic fuzzy set, and bipolar fuzzy set have been delivered. Notwithstanding, every hypothesis has inalienable difficulties.[10] Molodtsov started the idea of vague set hypothesis as another scientific device for managing uncertainties, which is liberated from the above mentioned impediments Maji et al. [1] offered the principal functional use of vague sets in Decision taking. These creators presented and examined the more summed up idea of the fuzzy vague set, which is a mix of fuzzy and vagues.[2] Majumdar and Samanta generalized the fuzzy vague set idea and Maji et al. [3] presented the concept of fuzzy vague set. Another sort of fuzzy set (multi-fuzzy set) was presented by [9] Sebastian and Ramakrishnan. This set uses requested successions of enrolment work and gives another technique to speak to issues not caught in different augmentations of fuzzy set hypothesis, such as pixel shading The idea of multi-fuzzy complex numbers and sets were first presented by [5].Dye and Pal These creators presented multi fuzzy Complex nilpotent frameworks over a distributive cross section. As of late, Yong et al. [4] proposed the idea of the multi-fuzzy vague set for its application, which is a more common fuzzy vague set. [6] Z. Kong, L. Gao, L. Wang, A they gave the theoretic approach in decision making problem[7] F. Feng, Y. Li, N. Cagman gave the concept of Generalized decision taking schemes based on choice value vague sets . [11] N. Anitha, gave the selection coefficient using interval value fuzzy vague set.[12].L.A Zadeh introduce the concept of fuzzy sets in 1965. [15] F.Feng,Y.B.Jun,X.Liu and L.Li and D.Yu present the concept of combination of interval valued fuzzy set and vague sets,[13] L.A.Zadeh, the concept of a linguistic variable and its application to approximate reasoning,[14] D.Chen, E.C.C. Tsang,D.S.Yeung and X.Wang gave the concept of the Parameterization reduction of vague set and its application The reason for this paper is to the idea fuzzy vague parameter set, and combination of fuzzy vague parameter set and beat existing uncertainties identified with this topic. After that the combination of fuzzy vague parameter set is more possible since it represents uncertainties in the selection of a fuzzy vague set compared to every factor value. Relations on combinations of fuzzy vague parameter sets are characterized and their properties are considered. A selection taking is then unravelled to recognize its common-sense applications. To encourage our conversation, we first surveyed some foundations on the vague set, fuzzy vague set, of fuzzy vague parameter set in Section 2. In Section 3, the idea of combination of fuzzy vague parameter set and operation is presented, and sure of its auxiliary properties are contemplated. In Section 4, the combination of fuzzy vague parameter set is utilized to dissect a selection taking,

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and a calculation is proposed. Finally, conclusion in Section 5.

1. Perimilaniry

All through this paper, C is common set (Universal set) E is a factor set , and $P(C)$ is the power set of C and $X \subseteq E$.

Definition 1(10). Vague set over $\mathbb{C}$ defined by a pair (F, X), where F is function given by $F: X \to P(C)$ in alternative a vague set over C is a functions from parameter to$P(C)$.$P(C)$ is not a set, but factor relations of subset of C

Example 1 Take $C = \{c_1, c_2, c_3, c_4, c_5\}$ is the set of different cars under certain conditions. Let $X = \{s_1, s_2, s_3\}$ is the set of parameter , here we considers$s_1 = costly$,$s_2 = good - looking$, $s_3 = better\ mileage$. Assume that $F(s_1) = \{c_2, c_4\}$,$F(s_2) = \{c_1, c_4, c_5\}$, $F(s_3) = \{c_1, c_3\}$.Therfore the vague set (F, X) find out ''pleasant appearance of the cars''. $F(s_1)$ Consider '' cars (Costly)'' whose mapping rate is the set$\{c_2, c_4\}$,$F(s_2)$ consider ''Cars (good –looking)''whose mapping rate is the set $\{s_1, s_4, s_5\}$ and $F(s_3)$ consider ''cars (better -mileage)''whose mapping rate is the set$\{s_1, s_3\}$.

Definition.2. (1) Let $\hat{P}(C)$ be all subset of common (universal) set C.A pair$(\hat{F}, X)$ is stated as fuzzy vague set over C. Here $\hat{F}$ is a function given by$\hat{F}: X \to \hat{P}(C)$.

Example. 2. The above example 1. The fuzzy vague set$(\hat{F}, X)$ can find out 'pleasant appearance of the cars' 'below the fuzzy condition.

$$\hat{F}(s_1) = \left\{\left(c_1, \frac{3}{10}\right), \left(c_2, \frac{8}{10}\right), \left(c_3, \frac{4}{10}\right), \left(c_4, \frac{7}{10}\right), \left(c_5, \frac{5}{10}\right)\right\}$$

$$\hat{F}(s_2) = \left\{\left(c_1, \frac{7}{10}\right), \left(c_2, \frac{2}{10}\right), \left(c_3, \frac{4}{10}\right), \left(c_4, \frac{8}{10}\right), \left(c_5, \frac{9}{10}\right)\right\}$$

$$\hat{F}(s_3) = \left\{\left(c_1, \frac{6}{10}\right), \left(c_2, \frac{4}{10}\right), \left(c_3, \frac{7}{10}\right), \left(c_4, \frac{2}{10}\right), \left(c_5, \frac{1}{10}\right)\right\}$$

Definition. 3. Let Q-be a non-negative integer. A Q-fuzzy vague parameter set $\hat{X}$ in C is a set

$$\hat{X} = \left\{c, (\mathcal{V}_1(c), \mathcal{V}_2(c), \dots \mathcal{V}_Q(c)\right) : c \in C\}, \quad \text{where} \mathcal{V}_j \in \hat{P}(C), j = 1,2,3, \dots Q.$$

Therefore the function $\mathcal{V}_{\hat{X}} = \left(\left(\mathcal{V}_1(c), \mathcal{V}_2(c), \dots \mathcal{V}_Q(c)\right)\right)$ is said to be Q −membership degree value of Q −fuzzy set $\hat{X}$ here Q is also dimension of$\hat{X}$. Therefore all the $Q -$ fuzzy sets of dimension Q in C denoted by $Q_Q SF(C)$.

Definition .4 Let$\hat{X} \in Q_Q FS(C)$. If $\hat{X} = \{(c, (0,0, \dots 0)) / c \in C\}$, then $\hat{X}$ is said to be empty Q −fuzzy set and its dimension Q. If $\hat{X} = \{(c, (1,1, \dots 1)) / c \in C\}$, therefore $\hat{X}$ is said to be identity $Q -$ fuzzy set and its dimensions is Q.

Definition.5. Let $\hat{X}$=$\{(c, \left(\mathcal{V}_1(c), \mathcal{V}_2(c), \dots, \mathcal{V}_Q(c)\right) / c \in C\}$ and

$\hat{Y} = \{(c, \left(\tau_1(c), \tau_2(c), \dots, \tau_K(c)\right) / c \in C$ be two $Q -$ fuzzy vague parameters set of C whose dimension Q. And we state the following operation can made.

(i) $\hat{X} \subseteq \hat{Y} \Leftrightarrow \mathcal{V}_j(c) \leq \tau_j(c)$, for all $c \in C, 1 \leq j \leq Q$.

(ii) $\hat{X} = \hat{Y} \Leftrightarrow \mathcal{V}_j(c) = \tau_j(c)$, for all $c \in C, 1 \leq j \leq Q$.

(iii) $\hat{X} \prod \hat{Y} = \{(c, \left(\mathcal{V}_1(c) \vee \tau_1(c), \mathcal{V}_2(c) \vee \tau_2(c), \dots, \mathcal{V}_Q(c) \vee \tau_Q(c)\right) / c \in C\}$

(iv) $\hat{X} \prod \hat{Y} = \{(c, \left(\mathcal{V}_1(c) \wedge \tau_1(c), \mathcal{V}_2(c) \wedge \tau_2(c), \dots, \mathcal{V}_Q(c) \wedge \tau_Q(c)\right) / c \in C\}$

(v) $\hat{X}' = \{(c, \left(\mathcal{V}_1'(c), \mathcal{V}_2'(c), \dots, \mathcal{V}_Q'(c)\right) / c \in C\}$

Definition.6. Let $(\hat{F}, X)$ is said to be $Q -$ fuzzy vague parameter set of dimension Q over C,wher $\hat{F}$ is function is given by $\hat{F}: X \to Q_Q SF(C)$.

A Q −fuzzy vague parameter set is a function from parameter to $K_K SF(C)$. then this set parameter family of Q −fuzzy subset of$C$. For$s \in X$, $\hat{F}(s)$ may be assume a set of $s \cong$ members of the $Q -$ fuzzy vague parameter set $(\hat{F}, X)$.

Example.3. Consider $C = \{m_1, m_2, m_3, m_4, m_5\}$ be the set of moter bike under circumentence. $X = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ is the factor set, here we consider ε_1 be the moter bike colur,which assign black, red,and yellow, ε_2 be the moter bike component , which can made it metal,fiber and plastic, and ε_3 be the moter bike cost which is low , medium, high. Now we state a $Q -$ fuzzy vague parameter set of three dimension as follows:

$$\hat{F}(\varepsilon_1) = \left\{\left(m_1, \left(\left(, \frac{2}{10}, \frac{3}{10}\right)\right), \left(m_2, \left(\frac{2}{10}, \frac{1}{10}, \frac{6}{10}\right)\right), \left(m_3, \left(\frac{1}{10}, \frac{3}{10}, \frac{4}{10}\right)\right), \left(m_4, \left(\frac{3}{10}, \frac{1}{10}, \frac{3}{10}\right)\right), \left(m_5, \left(\frac{7}{10}, \frac{1}{10}, \frac{2}{10}\right)\right)\right\}$$

$$\hat{F}(\varepsilon_2) = \left\{\left(m_1, \left(\frac{1}{10}, \frac{2}{10}, \frac{6}{10}\right), \left(m_2, \left(\frac{3}{10}, \frac{2}{10}, \frac{4}{10}\right)\right), \left(m_3, \left(\frac{1}{10}, \frac{3}{10}, \frac{4}{10}\right)\right), \left(m_4, \left(\frac{3}{10}, \frac{1}{10}, \frac{3}{10}\right)\right), \left(m_5, \left(\frac{7}{10}, \frac{1}{10}, \frac{2}{10}\right)\right)\right\}$$

$$\hat{F}(\varepsilon_3) = \left\{\left(m_1, \left(\frac{3}{10}, \frac{4}{10}, \frac{1}{10}\right), \left(m_2, \left(\frac{4}{10}, \frac{1}{10}, \frac{2}{10}\right)\right), \left(m_3, \left(\frac{2}{10}, \frac{2}{10}, \frac{5}{10}\right)\right), \left(m_4, \left(\frac{7}{10}, \frac{1}{10}, \frac{2}{10}\right)\right), \left(m_5, \left(\frac{5}{10}, \frac{2}{10}, \frac{3}{10}\right)\right)\right\}$$

Definition.7. Let $X, Y \subseteq E$. Let $(\hat{F}, X)$and $(\hat{G}, Y)$ be two $Q-$ fuzzy vague parameter sets wose dimensions Kover C.Therefoer $(\hat{F}, X)$ is called a $Q-$ fuzzy vague parameter subset of $(\hat{G}, Y)$if.

(i) $X \sqsubseteq Y$ and

(ii) $(\hat{F}(\varepsilon) \subseteq \hat{G}(\varepsilon), \forall \varepsilon \in X$.

Definition.8. A $Q-$ fuzzy vague parameter set $(\hat{F}, X)$ whose dimension Q over$C$ is called empty $Q-$ fuzzy vague parameter set if $\hat{F}(\varepsilon) = \hat{\emptyset}_Q, \forall \varepsilon \in X$.

Definition.9.A $Q-$ fuzzy vague parameter set $(\hat{F}, X)$ wose dimension K over C is called identity $Q-$ fuzzy vague parameter set if $\hat{F}(\varepsilon) = \hat{1}_Q, \forall \varepsilon \in X$.

2.Combination of Q −fuzzy vague parameter set

Definition.10. Let $C = \{\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \dots, \mathcal{X}_Q\}$ is the common universal set of elements and $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_Q\}$ is the common univesal set of parameter and $X \sqsubseteq E$. Let $(\hat{F}, X)$ is a $Q-$ fuzzy vague parameter set whose dimension Q over C, where $\hat{F}$ is a function give by $\hat{F}: X \to Q_Q SF(C)$. Moreover,let $\mathcal{V}$ be a fuzzy subset of X. therefore $\mathcal{V}: X \to [0,1]$. After , the pair,$(\hat{F}^{\mathcal{V}}, X)$ is said to be a combination of $Q-$ fuzzy vague parameter set whose dimension Q −over C, here $\hat{F}^{\mathcal{V}}$ is a function given by $\hat{F}^{\mathcal{V}}: X \to Q_Q SF(C) \otimes [0,1]$ along $\hat{F}^{\mathcal{V}}(\varepsilon) = \left(\hat{F}(\varepsilon), \mathcal{V}(\varepsilon)\right)$ and $\hat{F}(\varepsilon) \in Q_Q SF(C)$.

Here , for all factor ε_i,$\hat{F}^{\mathcal{V}}(\varepsilon_j) = \left(\hat{F}(\varepsilon_j), \mathcal{V}(\varepsilon_j)\right)$ indicates the values of $\in$ of the elements of C in $\hat{F}(\varepsilon_j)$ and of the possiblity of such $\in$.This parameter is presented by $\mathcal{V}(\varepsilon_i)$.

Example.4. Considre $C = \{m_1, m_2, m_3, m_4, m_5\}$ be the set of moter bike under concern, $X = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ be the set of paramete, here ε_1 be the moter bike colur,which assign black, red,and yellow, ε_2 be the moter bike component , which can made it metal,fiber and plastic, and ε_3 be the moter bike cost which is lo, medium, high.let $\mathcal{V}: X \to [0,1]$ is stated as $\mathcal{V}(\varepsilon_1) = \frac{2}{10}, \mathcal{V}(\varepsilon_2) = \frac{3}{10}, \mathcal{V}(\varepsilon_3) = \frac{6}{10}$.therefore we state a combination of $Q-$ fuzzy vague parameter set whose dimension is theree as follows.

$$\hat{F}^{\mathcal{V}}(\varepsilon_1) = \left\{\begin{pmatrix} m_1,\left(\left(\frac{4}{10},\frac{2}{10},\frac{3}{10}\right)\right),\left(m_2,\left(\frac{2}{10},\frac{1}{10},\frac{6}{10}\right)\right),,\left(m_3,\left(\frac{1}{10},\frac{3}{10},\frac{4}{10}\right)\right), \\ \left(m_4,\left(\frac{3}{10},\frac{1}{10},\frac{3}{10}\right)\right),\left(m_5,\left(\frac{7}{10},\frac{1}{10},\frac{2}{10}\right)\right) \end{pmatrix}\right\}$$

$$\hat{F}^{\mathcal{V}}(\varepsilon_2) = \left\{\begin{pmatrix} m_1,\left(\frac{1}{10},\frac{2}{10},\frac{6}{10}\right),\left(m_2,\left(\frac{3}{10},\frac{2}{10},\frac{4}{10}\right)\right),\left(m_3,\left(\frac{1}{10},\frac{3}{10},\frac{4}{10}\right)\right), \\ \left(m_4,\left(\frac{3}{10},\frac{1}{10},\frac{3}{10}\right)\right),\left(m_5,\left(\frac{7}{10},\frac{1}{10},\frac{2}{10}\right)\right) \end{pmatrix}\right\}$$

$$\hat{F}^{\mathcal{V}}(\varepsilon_2) = \left\{\begin{pmatrix} m_1,\left(\frac{3}{10},\frac{4}{10},\frac{1}{10}\right),\left(m_2,\left(\frac{4}{10},\frac{1}{10},\frac{2}{10}\right)\right),\left(m_3,\left(\frac{2}{10},\frac{2}{10},\frac{5}{10}\right)\right), \\ \left(m_4,\left(\frac{7}{10},\frac{1}{10},\frac{2}{10}\right)\right),\left(m_5,\left(\frac{5}{10},\frac{2}{10},\frac{3}{10}\right)\right) \end{pmatrix}\right\}$$

This can be represented in matrix form

$$\hat{F}^{\mathcal{V}} = \begin{bmatrix} \left(\frac{4}{10}\ \frac{2}{10}\ \frac{3}{10}\right) & \left(\frac{2}{10}\ \frac{1}{10}\ \frac{6}{10}\right) & \left(\frac{1}{10}\ \frac{3}{10}\ \frac{4}{10}\right) & \left(\frac{3}{10}\ \frac{1}{10}\ \frac{3}{10}\right) & \left(\frac{7}{10}\ \frac{1}{10}\ \frac{2}{10}\right) & \frac{2}{10} \\ \left(\frac{1}{10}\ \frac{2}{10}\ \frac{6}{10}\right) & \left(\frac{3}{10}\ \frac{2}{10}\ \frac{4}{10}\right) & \left(\frac{1}{10}\ \frac{3}{10}\ \frac{4}{10}\right) & \left(\frac{3}{10}\ \frac{1}{10}\ \frac{3}{10}\right) & \left(\frac{7}{10}\ \frac{1}{10}\ \frac{2}{10}\right) & \frac{3}{10} \\ \left(\frac{3}{10}\ \frac{4}{10}\ \frac{1}{10}\right) & \left(\frac{4}{10}\ \frac{1}{10}\ \frac{2}{10}\right) & \left(\frac{2}{10}\ \frac{2}{10}\ \frac{5}{10}\right) & \left(\frac{7}{10}\ \frac{1}{10}\ \frac{2}{10}\right) & \left(\frac{5}{10}\ \frac{2}{10}\ \frac{3}{10}\right) & \frac{6}{10} \end{bmatrix}$$

Here jth row value represents $\hat{F}^{\mathcal{V}}(\varepsilon_j)$, the jth colum value represents m_j, the last column value represents the value of $\mathcal{V}$ and this is the membership matrix of $\hat{F}^{\mathcal{V}}$.

Definition.11. Let $X, Y \subseteq E$. Let $(\hat{F}_{\mathcal{V}}, X)$ and $(\hat{G}_{\gamma}, Y)$ be two combination $Q-$ fuzzy vague parameter sets whose dimensions K over C. Now , $(\hat{F}_{\mathcal{V}}, X)$ is called a combination of Q −fuzzy vague parameter set of $(\hat{G}_{\gamma}, Y)$ if

(i) $(\hat{F}, X) \sqsubseteq (\hat{G}, B)$, and

(ii) $\mathcal{V}(\varepsilon) \leq \gamma(\varepsilon). \forall \varepsilon \in X$.

Also we represent $(\hat{F}_{\mathcal{V}}, X) \sqsubseteq (\hat{G}_{\gamma}, Y)$.

Example.5. Let $C = \{m_1, m_2, m_3, m_4, m_5\}$, $X = \{\varepsilon_1, \varepsilon_2\}$ and $Y = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$. Here we define the mapping $\mathcal{V}: X \to [0,1]$ as follws $\mathcal{V}(\varepsilon_1) = \frac{4}{10}$,$\mathcal{V}(\varepsilon_2) = \frac{3}{10}$ and $\gamma: Y \to [0,1]$ be stated as follows as $\gamma(\varepsilon_1) = \frac{6}{10}, \gamma(\varepsilon_2) = \frac{3}{10}, \gamma(\varepsilon_3) = \frac{5}{10}$.

Assume $(\hat{F}_{\mathcal{V}}, X)$ and $(\hat{G}_{\gamma}, Y)$ is two combination $Q-$ fuzzy vague parameter sets whose dimension three over C, stated as follows:

$$\hat{F}_{\mathcal{V}}(\varepsilon_1) = \left\{\begin{pmatrix} m_1,\left(\left(\frac{4}{10},\frac{2}{10},\frac{3}{10}\right)\right),\left(m_2,\left(\frac{2}{10},\frac{1}{10},\frac{6}{10}\right)\right),\left(m_3,\left(\frac{1}{10},\frac{3}{10},\frac{4}{10}\right)\right), \\ \left(m_4,\left(\frac{3}{10},\frac{1}{10},\frac{3}{10}\right)\right),\left(m_5,\left(\frac{7}{10},\frac{1}{10},\frac{2}{10}\right)\right) \end{pmatrix}, \frac{4}{10}\right\}$$

$$\hat{F}_{\mathcal{V}}(\varepsilon_2) = \left\{\begin{pmatrix} m_1,\left(\frac{1}{10},\frac{2}{10},\frac{6}{10}\right),\left(m_2,\left(\frac{3}{10},\frac{2}{10},\frac{4}{10}\right)\right),\left(m_3,\left(\frac{1}{10},\frac{3}{10},\frac{4}{10}\right)\right), \\ \left(m_4,\left(\frac{3}{10},\frac{1}{10},\frac{3}{10}\right)\right),\left(m_5,\left(\frac{7}{10},\frac{1}{10},\frac{2}{10}\right)\right) \end{pmatrix}, \frac{3}{10}\right\}$$

$$\hat{G}_{\gamma}(\varepsilon_1) = \left\{\begin{pmatrix} m_1,\left(\frac{5}{10},\frac{2}{10},\frac{6}{10}\right),\left(m_2,\left(\frac{2}{10},\frac{4}{10},\frac{8}{10}\right)\right),\left(m_3,\left(\frac{1}{10},\frac{4}{10},\frac{4}{10}\right)\right), \\ \left(m_4,\left(\frac{5}{10},\frac{4}{10},\frac{4}{10}\right)\right),\left(m_5,\left(\frac{7}{10},\frac{5}{10},\frac{2}{10}\right)\right) \end{pmatrix}, \frac{6}{10}\right\}$$

$$\hat{G}_{\gamma}(\varepsilon_2) = \left\{\begin{pmatrix} m_1,\left(\frac{3}{10},\frac{2}{10},\frac{9}{10}\right),\left(m_2,\left(\frac{4}{10},\frac{4}{10},\frac{4}{10}\right)\right),\left(m_3,\left(\frac{6}{10},\frac{4}{10},\frac{2}{10}\right)\right), \\ \left(m_4,\left(\frac{6}{10},\frac{1}{10},\frac{3}{10}\right)\right),\left(m_5,\left(\frac{6}{10},\frac{4}{10},\frac{2}{10}\right)\right) \end{pmatrix}, \frac{3}{10}\right\}$$

$\hat{G}_\gamma(\varepsilon_3) =$
$\left\{\left(\begin{matrix} m_1,\left(\frac{3}{10},\frac{4}{10},\frac{1}{10}\right),\left(m_2,\left(\frac{4}{10},\frac{1}{10},\frac{2}{10}\right)\right),\left(m_3,\left(\frac{2}{10},\frac{2}{10},\frac{5}{10}\right)\right), \\ \left(m_4,\left(\frac{7}{10},\frac{1}{10},\frac{2}{10}\right)\right),\left(m_5,\left(\frac{5}{10},\frac{2}{10},\frac{3}{10}\right)\right)\end{matrix}\right\},\frac{5}{10}\right\}.$

Definition.12. Let $X, Y \sqsubseteq E$. Let $(\hat{F}_\mathcal{V}, X)$ and $(\hat{G}_\gamma, Y)$ is combination $Q-$ fuzzy vague parameter sets whose dimension Q over C. $(\hat{F}_\mathcal{V}, X)$ and $(\hat{G}_\gamma, Y)$ are called combination of $Q-$ fuzzy vague parameter set equal if $(\hat{F}_\mathcal{V}, X)$ be a $Q-$ fuzzy vague parameter subset of $(\hat{G}_\gamma, Y)$ be a $Q-$ fuzzy vague parameter subset of $(\hat{F}_\mathcal{V}, X)$.

This can be written as $(\hat{F}_\mathcal{V}, X) \cong (\hat{G}_\gamma, Y)$.

Definition.13. A combination $Q-$ fuzzy vague parameter vague parameters setsset $(\hat{F}_\mathcal{V}, X)$ whose dimension Q over Q over C is called a combination empty $Q-$ fuzzy vague paramete set ,if $\hat{F}_\mathcal{V}(\varepsilon) = (\hat{F}(\varepsilon), \mathcal{V}(\varepsilon) = (\emptyset^K{}_{(\mathcal{V},X)}, 0), \forall \varepsilon \in X$.

Definition.14. A combination $Q-$ fuzzy vague parameter vague set $(\hat{F}_\mathcal{V}, X)$ whose dimension Q over C is called a combination identity $Q-$ fuzzy vague parameter set ,if $\hat{F}_\mathcal{V}(\varepsilon) = (\hat{F}(\varepsilon), \mathcal{V}(\varepsilon) = (C^Q{}_{(\mathcal{V},X)}, 1), \forall \varepsilon \in X$.

III.Combination of Q −fuzzy vague parameter set operations

Definition.15 The complement of a combination of Q −fuzzy vague parameter set $(\hat{F}_\mathcal{V}, X)$ whose dimension Q over C and it is deonted by $(\hat{F}_\mathcal{V}, X)\,'$ and is stated by $(\hat{F}_\mathcal{V}, X)\,' = (\hat{F}_{\mathcal{V}'}{}', X)$ where $\hat{F}_\mathcal{V}: X \to K_Q SF(C) \otimes 1$ is a function given by $\hat{F}_{\mathcal{V}'}{}'(\varepsilon) = \left(\left(\hat{F}(\varepsilon)', \mathcal{V}'(\varepsilon)\right)\right), \forall \varepsilon \in X$. Therefore $(\hat{F}_{\mathcal{V}'}{}')'$ be the similar as $\hat{F}_\mathcal{V}$ that is combination $Q-$ fuzzy vague parameter complement set. Consider $(\hat{\emptyset}_{(\mathcal{V},X)}{}^Q)$ and $\hat{C}_{(\mathcal{V},X)}{}^Q$ be the combination of $Q-$ fuzzy vague parameter set whose dimension Q over C,then $(\hat{\emptyset}_{(\mathcal{V},X)}{}^Q)' = \hat{C}_{(\mathcal{V},X)}{}^Q$ and $(\hat{C}_{(\mathcal{V},X)}{}^Q)'$ and $\hat{\emptyset}_{(\mathcal{V},X)}{}^Q$.

Example .6 Consider example 4 we have $(\hat{F}_\mathcal{V}, X)'$ as follws

$$\hat{F}_{\mathcal{V}'}{}'(\varepsilon_1) = (\{\, m_1, \left(\frac{6}{10},\frac{8}{10},\frac{7}{10}\right), (\, m_2, \left(\frac{8}{10},\frac{9}{10},\frac{7}{10}\right), \left(m_3, \left(\frac{9}{10},\frac{7}{10},\frac{6}{10}\right), \left(\, m_4, \left(\frac{7}{10},\frac{8}{10},\frac{6}{10}\right), \left(\, m_5, \left(\frac{5}{10},\frac{9}{10},\frac{8}{10}\right)\right\}, \frac{8}{10}\right\}\right)$$

$$\hat{F}_{\mathcal{V}'}{}'(\varepsilon_2) = (\{\, m_1, \left(\frac{9}{10},\frac{8}{10},\frac{4}{10}\right), (\, m_2, \left(\frac{7}{10},\frac{8}{10},\frac{6}{10}\right), \left(m_3, \left(\frac{5}{10},\frac{7}{10},\frac{9}{10}\right), \left(\, m_4, \left(\frac{4}{10},\frac{9}{10},\frac{7}{10}\right), \left(\, m_5, \left(\frac{4}{10},\frac{8}{10},\frac{9}{10}\right)\right\}, \frac{7}{10}\right\}\right)$$

$$\hat{F}_{\mathcal{V}'}{}'(\varepsilon_3) = (\{\, m_1, \left(\frac{7}{10},\frac{6}{10},\frac{9}{10}\right), (\, m_2, \left(\frac{6}{10},\frac{9}{10},\frac{8}{10}\right), \left(m_3, \left(\frac{8}{10},\frac{8}{10},\frac{5}{10}\right), \left(\, m_4, \left(\frac{3}{10},\frac{9}{10},\frac{8}{10}\right), \left(\, m_5, \left(\frac{5}{10},\frac{8}{10},\frac{7}{10}\right)\right\}, \frac{4}{10}\right\}\right)$$

, this can br representet in matrix form

$$\hat{F}_{\mathcal{V}'}{}' = \begin{bmatrix} \left(\frac{6}{10} & \frac{8}{10} & \frac{7}{10}\right) & \left(\frac{8}{10} & \frac{9}{10} & \frac{4}{10}\right) & (\frac{9}{10} & \frac{7}{10} & \frac{6}{10}) & (\frac{7}{10} & \frac{9}{10} \\ (\frac{9}{10} & \frac{8}{10} & \frac{4}{10}) & (\frac{7}{10} & \frac{8}{10} & \frac{6}{10}) & (\frac{5}{10} & \frac{7}{10} & \frac{9}{10}) & (\frac{4}{10} & \frac{9}{10} \\ (\frac{7}{10} & \frac{6}{10} & \frac{9}{10}) & (\frac{6}{10} & \frac{9}{10} & \frac{8}{10}) & (\frac{8}{10} & \frac{8}{10} & \frac{5}{10}) & (\frac{3}{10} & \frac{9}{10} \end{bmatrix}$$

Next, we represent the notation of $''\wedge(AND)''$ (infimum) and $''\vee(OR)''$(suprimum) operator on combination of Q −fuzzy vague parameter set.

Definition.16. Let $(\hat{F}_\mathcal{V}, X)$ and $(\hat{F}_\gamma, Y)$ be two combination of Q −fuzzy vague parameter sets whose dimension Q over$C$, then $''\wedge(AND)''$ stated as $(\hat{F}_\mathcal{V}, X)\wedge^{inf} (\hat{F}_\gamma, Y) = (\widehat{\mathcal{M}}_\tau, X \otimes Y)$, where $\widehat{\mathcal{M}}(\ell, m) = \hat{F}(\ell) \Cap \hat{G}(m)$ and $\tau(\ell, m) = \mathcal{V}(\ell)\wedge^{min}\gamma(m), \forall\, \ell, m \in X \otimes Y$.

Definition.17. Let $(\hat{F}_\mathcal{V}, X)$ and $(\hat{F}_\gamma, Y)$ be two combination of Q −fuzzy vague parameter sets whose dimension Q over C, then $''\vee(OR)''$ stated as $(\hat{F}_\mathcal{V}, X)\vee^{sup} (\hat{F}_\gamma, Y) = (\widehat{\mathcal{N}}_\tau, X \otimes Y)$, where $\widehat{\mathcal{N}}(\ell, m) = \hat{F}(\ell) \Cup \hat{G}(m)$ and $\tau(\ell, m) = \mathcal{V}(\ell)\vee^{sup}\gamma(m), \forall\, \ell, m \in X \otimes Y$.

Example 8. Let $C = \{m_1, m_2, m_3\}$, $X = \{\varepsilon_1, \varepsilon_2\}$ and $B = \{\varepsilon_2, \varepsilon_3, \varepsilon_4\}$. and define the mapping $\mathcal{V}: X \to [0,1]$ and stated as $\mathcal{V}(\varepsilon_1) = \frac{4}{10}, \mathcal{V}(\varepsilon_2) = \frac{3}{10}$ and $\gamma: Y \to [0,1]$ and stated as $\gamma(\varepsilon_2) = \frac{6}{10}, \gamma(\varepsilon_3) = \frac{2}{10}, \gamma(\varepsilon_4) = \frac{5}{10}$. Assume $(\hat{F}_{\mathcal{V},}X)$ and$(\hat{G}_\gamma, Y)$ be combination Q −fuzzy vague parameter set whose dimension three over C, stated as follows :

$\hat{F}_\mathcal{V}(\varepsilon_1) =$
$\left(\left\{(m_1, \left(\frac{4}{10},\frac{7}{10},\frac{5}{10}\right), (m_2, \left(\frac{4}{10},\frac{7}{10},\frac{5}{10}\right), (m_3, \left(\frac{4}{10},\frac{7}{10},\frac{5}{10}\right)\right\}, \frac{4}{10}\right)$
,
$\hat{F}_\mathcal{V}(\varepsilon_2) =$
$\left(\left\{(m_1, \left(\frac{6}{10},\frac{2}{10},\frac{6}{10}\right), (m_2, \left(\frac{3}{10},\frac{9}{10},\frac{4}{10}\right), (m_3, \left(\frac{5}{10},\frac{3}{10},\frac{4}{10}\right)\right\}, \frac{3}{10}\right)$
,
$\hat{G}_\gamma(\varepsilon_2) =$
$\left(\left\{(m_1, \left(\frac{3}{10},\frac{2}{10},\frac{9}{10}\right), (m_2, \left(\frac{4}{10},\frac{4}{10},\frac{4}{10}\right), (m_3, \left(\frac{6}{10},\frac{4}{10},\frac{2}{10}\right)\right\}, \frac{2}{10}\right),$

$\hat{G}_\gamma(\varepsilon_3) =$
$\left(\left\{(m_1, \left(\frac{5}{10},\frac{2}{10},\frac{6}{10}\right), (m_2, \left(\frac{2}{10},\frac{4}{10},\frac{8}{10}\right), (m_3, \left(\frac{1}{10},\frac{4}{10},\frac{4}{10}\right)\right\}, \frac{6}{10}\right)$
,

$\hat{G}_\gamma(\varepsilon_4) =$
$\left(\left\{(m_1, \left(\frac{6}{10},\frac{4}{10},\frac{9}{10}\right), (m_2, \left(\frac{4}{10},\frac{1}{10},\frac{5}{10}\right), (m_3, \left(\frac{2}{10},\frac{2}{10},\frac{5}{10}\right)\right\}, \frac{5}{10}\right)$
.

Therefore $(\hat{F}_\mathcal{V}, X)\wedge^{inf} (\hat{F}_\gamma, Y) = (\widehat{\mathcal{M}}_\tau, X \otimes Y)$ and $(\hat{F}_\mathcal{V}, X)\vee^{sup} (\hat{F}_\gamma, Y) = (\widehat{\mathcal{N}}_\tau, X \otimes Y)$. as follows

$\widehat{\mathcal{M}}_\tau(\varepsilon_1, \varepsilon_2) =$
$(\{m_1, \left(\frac{3}{10}, \frac{2}{10}, \frac{5}{10}\right), \left(m_1, \left(\frac{2}{10}, \frac{1}{10}, \frac{4}{10}\right), \left(m_1, \left(\frac{1}{10}, \frac{3}{10}, \frac{2}{10}\right)\right\}, \frac{2}{10}\right)\})$
,

$\widehat{\mathcal{M}}_\tau(\varepsilon_1, \varepsilon_3) =$
$\left(\left\{m_1, \left(\frac{4}{10}, \frac{2}{10}, \frac{5}{10}\right), \left(m_1, \left(\frac{2}{10}, \frac{1}{10}, \frac{8}{10}\right), \left(m_1, \left(\frac{1}{10}, \frac{3}{10}, \frac{4}{10}\right)\right\}, \frac{4}{10}\right)\right\}\right),$

$\widehat{\mathcal{M}}_\tau(\varepsilon_1, \varepsilon_4) =$
$\left(\left\{m_1, \left(\frac{4}{10}, \frac{4}{10}, \frac{5}{10}\right), \left(m_1, \left(\frac{2}{10}, \frac{1}{10}, \frac{5}{10}\right), \left(m_1, \left(\frac{1}{10}, \frac{2}{10}, \frac{5}{10}\right)\right\}, \frac{4}{10}\right)\right\}\right),$

$\widehat{\mathcal{M}}_\tau(\varepsilon_2, \varepsilon_2) =$
$\left(\left\{m_1, \left(\frac{3}{10}, \frac{2}{10}, \frac{6}{10}\right), \left(m_1, \left(\frac{3}{10}, \frac{4}{10}, \frac{4}{10}\right), \left(m_1, \left(\frac{1}{10}, \frac{3}{10}, \frac{4}{10}\right)\right\}, \frac{3}{10}\right)\right\}\right),$

$\widehat{\mathcal{M}}_\tau(\varepsilon_2, \varepsilon_3) =$
$\left(\left\{m_1, \left(\frac{5}{10}, \frac{2}{10}, \frac{6}{10}\right), \left(m_1, \left(\frac{3}{10}, \frac{1}{10}, \frac{4}{10}\right), \left(m_1, \left(\frac{2}{10}, \frac{2}{10}, \frac{4}{10}\right)\right\}, \frac{3}{10}\right)\right\}\right),$
And

$\widehat{\mathcal{N}}_\tau(\varepsilon_1, \varepsilon_2) =$
$\left(\left\{m_1, \left(\frac{4}{10}, \frac{7}{10}, \frac{9}{10}\right), \left(m_1, \left(\frac{4}{10}, \frac{4}{10}, \frac{8}{10}\right), \left(m_1, \left(\frac{6}{10}, \frac{4}{10}, \frac{8}{10}\right)\right\}, \frac{4}{10}\right)\right\}\right),$

$\widehat{\mathcal{N}}_\tau(\varepsilon_1, \varepsilon_3) =$
$\left(\left\{m_1, \left(\frac{5}{10}, \frac{7}{10}, \frac{6}{10}\right), \left(m_1, \left(\frac{2}{10}, \frac{4}{10}, \frac{8}{10}\right), \left(m_1, \left(\frac{1}{10}, \frac{4}{10}, \frac{8}{10}\right)\right\}, \frac{6}{10}\right)\right\}\right),$

$\widehat{\mathcal{N}}_\tau(\varepsilon_1, \varepsilon_4) =$
$\left(\left\{m_1, \left(\frac{6}{10}, \frac{7}{10}, \frac{9}{10}\right), \left(m_1, \left(\frac{4}{10}, \frac{1}{10}, \frac{8}{10}\right), \left(m_1, \left(\frac{2}{10}, \frac{3}{10}, \frac{8}{10}\right)\right\}, \frac{5}{10}\right)\right\}\right),$

$\widehat{\mathcal{N}}_\tau(\varepsilon_2, \varepsilon_2) =$
$\left(\left\{m_1, \left(\frac{6}{10}, \frac{2}{10}, \frac{9}{10}\right), \left(m_1, \left(\frac{3}{10}, \frac{9}{10}, \frac{8}{10}\right), \left(m_1, \left(\frac{5}{10}, \frac{4}{10}, \frac{4}{10}\right)\right\}, \frac{6}{10}\right)\right\}\right),$

$\widehat{\mathcal{N}}_\tau(\varepsilon_2, \varepsilon_3) =$
$\left(\left\{m_1, \left(\frac{6}{10}, \frac{2}{10}, \frac{6}{10}\right), \left(m_1, \left(\frac{3}{10}, \frac{9}{10}, \frac{8}{10}\right), \left(m_1, \left(\frac{5}{10}, \frac{4}{10}, \frac{4}{10}\right)\right\}, \frac{6}{10}\right)\right\}\right),$

$\widehat{\mathcal{N}}_\tau(\varepsilon_2, \varepsilon_4) =$
$\left(\left\{m_1, \left(\frac{6}{10}, \frac{4}{10}, \frac{9}{10}\right), \left(m_1, \left(\frac{4}{10}, \frac{9}{10}, \frac{5}{10}\right), \left(m_1, \left(\frac{5}{10}, \frac{3}{10}, \frac{5}{10}\right)\right\}, \frac{5}{10}\right)\right\}\right).$

Proportion 1. Let $(\hat{F}_\mathcal{V}, X)$ and $(\hat{F}_\gamma, Y)$ is two combination of $Q-$ fuzzy vague parameter sets whose dimension Q over C. then

(i) $((\hat{F}_\mathcal{V}, X)\wedge^{inf}(\hat{G}_\gamma, Y))' = (\hat{F}_\mathcal{V}, X)'\vee^{suf}(\hat{G}_\gamma, Y)'$

(ii) $((\hat{F}_\mathcal{V}, X)\vee^{inf}(\hat{G}_\gamma, Y)' = (\hat{F}_\mathcal{V}, X)'\wedge^{suf}(\hat{G}_\gamma, Y)'$.

Proof.

(i) Let $(\hat{F}_\mathcal{V}, A)\wedge^{inf}(\hat{G}_\gamma, Y) = (\widehat{\mathcal{M}}_\tau, X \otimes Y)$.

Since, $((\hat{F}_\mathcal{V}, X)\wedge^{inf}(\hat{G}_\gamma, Y))' = (\widehat{\mathcal{M}}_\tau, X \otimes Y)' = \mathcal{M}_{\tau'}{}', X \otimes Y)$, where
$\tau'(a,b) = (\mathcal{V}\wedge^{inf}\gamma)'(a,b)$
$= \mathcal{V}'(a)\vee^{suf}\gamma'(b), \forall a, b \in X \otimes Y.$
This implies $(\hat{F}_\mathcal{V}, X)'\vee^{sup}(\hat{G}_\gamma, Y)' = (\hat{F}_{\mathcal{V}'}{}', X)\vee^{suf}(\hat{G}_{\mathcal{V}'}{}', Y) = (\widehat{\mathcal{N}}_\tau, X \otimes Y)$, where
$\tau(a,b) = \mathcal{V}'(a)\vee^{sup}\gamma'(b) = \tau'(a,b)\ \forall\ a, b \in X \otimes Y$. Hence

$\mathcal{M}_{\tau'}{}'(a,b) = (\widehat{\mathcal{M}}'(a,b), \tau'(a,b))$

$= ((\hat{F}(a) \Cap \hat{G}(b))'(a,b), \tau'(a,b))$

$= \left(\left(\hat{F}'(a) \Cup \hat{G}'(b)\right)(a,b), \tau'(a,b)\right)$

$= \widehat{\mathcal{N}}(a,b), \tau(a,b))$

$= (\widehat{\mathcal{N}}_\tau, X \otimes Y)$

Therefore $((\hat{F}_\mathcal{V}, X)\wedge^{min}(\hat{G}_\gamma, Y))' = (\hat{F}_\mathcal{V}, X)'\vee^{suf}(\hat{G}_\gamma, Y)'$

(ii) Let $(\hat{F}_\mathcal{V}, A)\vee^{suf}(\hat{G}_\gamma, Y) = (\widehat{\mathcal{M}}_\tau, X \otimes Y)$.

Since, $((\hat{F}_\mathcal{V}, X)\vee^{suf}(\hat{G}_\gamma, Y))' = (\widehat{\mathcal{M}}_\tau, X \otimes Y)' = \mathcal{M}_{\tau'}{}', X \otimes Y)$, where
$\tau'(a,b) = (\mathcal{V}\vee^{suf}\gamma)'(a,b)$
$= \mathcal{V}'(a)\wedge^{min}\gamma'(b), \forall a, b \in X \otimes Y.$
This implies $(\hat{F}_\mathcal{V}, X)'\wedge^{suf}(\hat{G}_\gamma, Y)' = (\hat{F}_{\mathcal{V}'}{}', X)\wedge^{suf}(\hat{G}_{\mathcal{V}'}{}', Y) = (\widehat{\mathcal{N}}_\tau, X \otimes Y)$, where
$\tau(a,b) = \mathcal{V}'(a)\wedge^{inf}\gamma'(b) = \tau'(a,b)\ \forall\ a, b \in X \otimes Y$. Hence

$\mathcal{M}_{\tau'}{}'(a,b) = (\widehat{\mathcal{M}}'(a,b), \tau'(a,b))$

$= ((\hat{F}(a) \Cup \hat{G}(b))'(a,b), \tau'(a,b))$

$= \left(\left(\hat{F}'(a) \Cap \hat{G}'(b)\right)(a,b), \tau'(a,b)\right)$

$= \widehat{\mathcal{N}}(a,b), \tau(a,b))$

$= (\widehat{\mathcal{N}}_\tau, X \otimes Y)$

Therefore $((\hat{F}_\mathcal{V}, X)\vee^{suf}(\hat{G}_\gamma, Y))' = (\hat{F}_\mathcal{V}, X)'\wedge^{inf}(\hat{G}_\gamma, Y)'$. Hence the theorem. //

IV.Combination of $Q-$ fuzzy vague parameter sets and its decision taking

Roy et al. obtainable an item identification algorithm based on a fuzzy vague parameter set. Made to order Roy's condition to match up to selection values of unlike things that are maximum selection value. [7] Feng et al. planned that the idea of selection values is intended for ordinary vague sets and is out of condition to find decision-taking problems connecting fuzzy vague parameter sets. Ref [8] represented a new concept to multi-fuzzy vague set based decision taking problems to apply Feng's condition. Here we convert to $Q-$ fuzzy vague parameter sets based on the above condition.

Let $C=\{A_1,A_2,\dots,A_Q\}$ is the common set of members, $E=\{\varepsilon_1,\varepsilon_2,\dots,\varepsilon_K\}$ is the common set of parameter and $X\subseteq E$. Let $(\hat{F}_{\gamma_1},X)$ is a combination of $Q-$ fuzzy vague parameter set whose dimension Q over C. for $\forall\varepsilon\in X$, $\hat{F}_{\gamma_1}(\varepsilon)=$

$(\{A_1,((\mathcal{V}_1(A_1),\mathcal{V}_2(A_1),\dots,\mathcal{V}_Q(A_1)),(A_2,(\mathcal{V}_1(A_2),\mathcal{V}_2(A_2),\dots,\mathcal{V}_Q(A_2)),\dots,$
$(A_Q,(\mathcal{V}_1(A_Q),\mathcal{V}_2(A_Q),\dots,\mathcal{V}_Q(A_Q))\},$

$\gamma_1(\varepsilon)=(\hat{F}(\varepsilon),\gamma_1(\varepsilon))$, Where $\hat{F}(\varepsilon)$ is a normalized $Q-$ fuzzy vague parameter set whose dimension $Q-$ over C.

Now,

$$\hat{F}(\varepsilon)=\begin{bmatrix}\mathcal{V}_1(A_1) & \mathcal{V}_2(A_1)\dots & \mathcal{V}_K(A_1)\\ \mathcal{V}_1(A_2) & \mathcal{V}_2(A_2)\dots & \mathcal{V}_K(A_2)\\ \dots. & \dots & \dots\\ \mathcal{V}_1(A_Q) & \mathcal{V}_2(A_Q) & \mathcal{V}_Q(A_Q)\end{bmatrix}$$

$$\hat{F}_{\gamma_1}(\varepsilon)=\left[\begin{bmatrix}\mathcal{V}_1(A_1) & \mathcal{V}_2(A_1)\dots & \mathcal{V}_K(A_1)\\ \mathcal{V}_1(A_2) & \mathcal{V}_2(A_2)\dots & \mathcal{V}_K(A_2)\\ \dots. & \dots & \dots\\ \mathcal{V}_1(A_Q) & \mathcal{V}_2(A_Q) & \mathcal{V}_Q(A_Q)\end{bmatrix},\gamma_1(\varepsilon)\right]$$

Assume $\mathcal{W}_{\gamma_2}(\varepsilon)\left(\begin{bmatrix}\mathcal{W}_1\\ \mathcal{W}_2\\ \cdot\\ \vdots\\ \mathcal{W}_Q\end{bmatrix},\gamma_2(\varepsilon)\right)$

is the relative (Weight) value of the factor ε.

Table (i) Combination of $K-$vague parameters setsset tabular reperesentation $(\hat{F}_{\gamma_1\gamma_2},X)$

$$\varepsilon_1\mathcal{W}_{\gamma_2}(\varepsilon_1)\left(\begin{bmatrix}\frac{3}{10}\\ \frac{4}{10}\\ \frac{3}{10}\end{bmatrix}\right),\varepsilon_2\mathcal{W}_{\gamma_2}(\varepsilon_2)\left(\begin{bmatrix}\frac{5}{10}\\ \frac{4}{10}\\ \frac{1}{10}\end{bmatrix}\right),\varepsilon_3\mathcal{W}_{\gamma_2}(\varepsilon_3)\left(\begin{bmatrix}\frac{2}{10}\\ \frac{3}{10}\\ \frac{5}{10}\end{bmatrix}\right),$$

t_1	$\left(\frac{293}{1000},\frac{171}{1000}\right)$	$\left(\frac{299}{1000},\frac{159}{1000}\right)$	$\left(\frac{296}{1000},\frac{179}{1000}\right)$
t_2	$\left(\frac{334}{1000},\frac{171}{1000}\right)$	$\left(\frac{409}{1000},\frac{159}{1000}\right)$	$\left(\frac{248}{1000},\frac{179}{1000}\right)$
t_3	$\left(\frac{270}{1000},\frac{171}{1000}\right)$	$\left(\frac{362}{1000},\frac{159}{1000}\right)$	$\left(\frac{236}{1000},\frac{179}{1000}\right)$
t_4	$\left(\frac{323}{1000},\frac{171}{1000}\right)$	$\left(\frac{268}{1000},\frac{159}{1000}\right)$	$\left(\frac{315}{1000},\frac{179}{1000}\right)$
t_5	$\left(\frac{302}{1000},\frac{171}{1000}\right)$	$\left(\frac{369}{1000},\frac{159}{1000}\right)$	$\left(\frac{320}{1000},\frac{179}{1000}\right)$

Table(ii)

The half stage fuzzy vague parameter set $H(\boxminus\hat{F}_{\gamma_1\gamma_2}:half)of\ \boxminus\hat{F}_{\gamma_1\gamma_2}$ along with selcction values

C	$\varepsilon_1\gamma_1(\varepsilon_1)\gamma_2(\varepsilon_1)=\frac{171}{1000}$	$\varepsilon_2\gamma_1(\varepsilon_1)\gamma_2(\varepsilon_2)=\frac{159}{1000}$	$\varepsilon_3\gamma_1(\varepsilon_1)\gamma_2(\varepsilon_3)=\frac{179}{1000}$	Sclection value
t_1	0	0	1	$s_1=1$
t_2	1	1	0	$s_2=2$
t_3	0	1	0	$s_3=1$
t_4	1	0	1	$s_4=2$
t_5	0	1	1	$s_5=2$

The half stage fuzzy vague parameter set $H(\boxminus\hat{F}_{\gamma_1\gamma_2}:half)of\ \boxminus\hat{F}_{\gamma_1\gamma_2}$ along with weighted selcction values.

C	ε_1	ε_2	ε_3	Weighted Sclection value
t_1	0	0	1	$\bar{s}_1=\frac{179}{1000}$
t_2	1	1	0	$\bar{s}_2=\frac{330}{1000}$
t_3	0	1	0	$\bar{s}_3=\frac{159}{1000}$
t_4	1	0	1	$\bar{s}_4=\frac{350}{1000}$
t_5	0	1	1	$\bar{s}_5=\frac{338}{1000}$

Now, we state an induced combination of $Q-$ fuzzy vague parameter set as:

$$\hat{F}_{\gamma_1\gamma_2}(\varepsilon)=\left(\begin{pmatrix}\mathcal{V}_1(A_1) & \mathcal{V}_2(A_1)\dots & \mathcal{V}_Q(A_1)\\ \mathcal{V}_1(A_2) & \mathcal{V}_2(A_2)\dots & \mathcal{V}_Q(A_2)\\ \dots. & \dots & \dots\\ \mathcal{V}_1(A_Q) & \mathcal{V}_2(A_Q) & \mathcal{V}_Q(A_Q)\end{pmatrix}\begin{pmatrix}\mathcal{W}_1\\ \mathcal{W}_2\\ \cdot\\ \vdots\\ \mathcal{W}_Q\end{pmatrix},\gamma_1(\varepsilon)\gamma_2(\varepsilon)\right)$$

$$=\left(\begin{pmatrix}\sum_{j=1}^{K}\mathcal{W}_j\mathcal{V}_j(A_1)\\ \sum_{j=1}^{K}\mathcal{W}_j\mathcal{V}_j(A_1)\\ \vdots\\ \sum_{j=1}^{K}\mathcal{W}_j\mathcal{V}_j(A_1)\end{pmatrix},\gamma_1(\varepsilon)\gamma_2(\varepsilon)\right)$$

Therefore , if $\mathcal{W}_{\gamma_2}(\varepsilon)$ is given, we can modify a combination of $Q-$ fuzzy vague parameter fraction set on factor set $\hat{F}_{\gamma_1}(\varepsilon)$ into an induced combination of $Q-$ fuzzy vague parameter set $\hat{F}_{\gamma_1\gamma_2}(\varepsilon)$. Therefore, we use induced combination of fuzzy vague parameter set $\hat{F}_{\gamma_1\gamma_2}(\varepsilon)$ to produce a selection taking.

IV.(i) An working rule

Enter : The combination $Q-$fuzzy vague parameter set $(\hat{F}_{\gamma_1}, X)$ and akin weight $\mathcal{W}(\varepsilon_j), \gamma_2(\varepsilon_j)$ of parameters setsε_j.

Output :The multiplication of A_Q for a few Q

Rule .1. A combination of $Q-$ fuzzy vague parameter set $(\hat{F}_{\gamma_1}A)$ and akin weight $\left(\mathcal{W}(\varepsilon_j), \gamma_2(\varepsilon_j)\right)$ of parameters setsε_j is an enter.

Rule .2. modify the $Q-$ fuzzy vague parameter set $(\hat{F}, X)$ into the ordinary $K-$ fuzzy vague parameter set.

Rule.3. Find the induced combination of fuzzy vague parameter set $\boxminus_{\hat{F}_{\gamma_1\gamma_2}} = (\hat{F}_{\gamma_1\gamma_2}, X)$

Rule .4. Select the half –level selection rule for selection taking.

Rule .5. Find the half-level fraction vague set $H\left(\boxminus \hat{F}_{\gamma_1\gamma_2}{:}\,half\right)$.

Rule.6. current the half-level vague fraction vague set $H\left(\boxminus \hat{F}_{\gamma_1\gamma_2}{:}\,half\right)$ in tabular form and find the selection value s_j of A_j for all j.

Rule 7.Find $\sup_j s_j$.

Rule.8.If superimum reached at one value after the best selection is to choose A_i.

Rule.9. If superimum reached for product value of j , after find the selection. weighted values $\overline{s}_i$ of A_i $\forall\, i$ and current the selection weighted values $\overline{s}_i$ tabular form.

Rule.10. Find superimum selected wight vlaue $\overline{s}_K = \sup_j\{\overline{s}_i\}$.

Rule .11. If Q has one and only value , afte the curréct selection is to chose $A_{Q.}$

Rule 12. If Q has greter than 1 value after any 1 of A_K possibly an correct selection.

Unmetrical Example

Consider $C = \{m_1, m_2, m_3, m_4, m_5\}$ be the set of moter bike under circumentence. $X = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ is the factor set, here we consider ε_1 be the moter bike colour,which assign black, red,and yellow, ε_2 be the moter bike component , which can made it metal,fiber and plastic, and ε_3 be the moter bike cost which is low , medium, high. Now we state a $Q-$fuzzy vague parameter set of three dimension as follows:

$$\hat{F}_{\gamma_1}(\varepsilon_1) = \left\{\begin{pmatrix} m_1,\left(\left(\frac{34}{100},\frac{32}{100},\frac{21}{100}\right)\right),\left(m_2,\left(\frac{40}{100},\frac{37}{100},\frac{22}{100}\right)\right),\left(m_3,\left(\frac{51}{100},\frac{12}{100},\frac{23}{100}\right)\right) \\ ,\left(m_4,\left(\frac{41}{100},\frac{41}{100},\frac{12}{100}\right)\right),\left(m_5,\left(\frac{35}{100},\frac{35}{100},\frac{19}{100}\right),\frac{45}{100}\right) \end{pmatrix}\right\}$$

$$\hat{F}_{\gamma_1}(\varepsilon_2) = \left\{\begin{pmatrix} m_1,\left(\left(\frac{24}{100},\frac{35}{100},\frac{39}{100}\right)\right),\left(m_2,\left(\frac{40}{100},\frac{37}{100},\frac{22}{100}\right)\right),\left(m_3,\left(\frac{57}{100},\frac{28}{100},\frac{12}{100}\right)\right) \\ ,\left(m_4,\left(\frac{41}{100},\frac{41}{100},\frac{12}{100}\right)\right),\left(m_5,\left(\frac{35}{100},\frac{35}{100},\frac{19}{100}\right),\frac{43}{100}\right) \end{pmatrix}\right\}$$

$$\hat{F}_{\gamma_1}(\varepsilon_3) = \left\{\begin{pmatrix} m_1,\left(\left(\frac{43}{100},\frac{25}{100},\frac{27}{100}\right)\right),\left(m_2,\left(\frac{25}{100},\frac{26}{100},\frac{24}{100}\right)\right),\left(m_3,\left(\frac{79}{100},\frac{11}{100},\frac{9}{100}\right)\right) \\ ,\left(m_4,\left(\frac{15}{100},\frac{45}{100},\frac{30}{100}\right)\right),\left(m_5,\left(\frac{35}{100},\frac{35}{100},\frac{29}{100}\right),\frac{46}{100}\right) \end{pmatrix}\right\}$$

Assume a customer like to choose a motor cycle depending on its show and the customer has compulsorily the following weights for the parameters sets in X, for the parameters sets bike colour, $\mathcal{W}_{\gamma_1}(\varepsilon_1) = \left(\begin{pmatrix}\frac{3}{10}\\ \frac{4}{10}\\ \frac{3}{10}\end{pmatrix}, \frac{38}{100}\right)$, and the parameters sets bike component,

$$\mathcal{W}_{\gamma_2}(\varepsilon_2) = \left(\begin{pmatrix}\frac{5}{10}\\ \frac{4}{10}\\ \frac{1}{10}\end{pmatrix}, \frac{37}{100}\right)$$

, and the parameters sets bike cost

$$\mathcal{W}_{\gamma_2}(\varepsilon_2) = \left(\begin{pmatrix}\frac{2}{10}\\ \frac{3}{10}\\ \frac{5}{10}\end{pmatrix}, \frac{39}{100}\right)$$

Therefore, the combination of fuzzy vague parameter set $\boxminus_{\hat{F}_{\gamma_1\gamma_2}} = (\hat{F}_{\gamma_1\gamma_2}, X)$ which shows table (i)

Therefore here we apply half –level decision rule, Hence

$$\widehat{half\ \boxminus}_{\hat{F}_{\gamma_1\gamma_2}} = \left\{\left(\varepsilon_1, \frac{3044}{10000}\right), \left(\varepsilon_2, \frac{3414}{10000}\right), \left(\varepsilon_3\, \frac{2830}{10000}\right)\right\}.$$

Therefore we can find the half –level vague set $H\left(\boxminus \hat{F}_{\gamma_1\gamma_2}{:}\,half\right)$ of $\boxminus \hat{F}_{\gamma_1\gamma_2}$ with selection value with table (ii) from table (ii) the selection values s_2, s_4, s_5 value is two is greater than s_1, s_3 is one. Hence $s_i = \sup\{s_1, s_2, s_3, s_4, s_5\}$ which is equal to s_2 or s_4 or s_5. thus the superimum value is exist for product values, and we have find the weighted selection value $\overline{s}_i$ of m_i $\forall\, i$. Now from table (iii) we have

(1) Weighted selection value for $m_1 = \bar{s}_1 = \left(0 \otimes \frac{1710}{10000}\right) + \left(0 \otimes \frac{1591}{10000}\right) + \left(1 \otimes \frac{1794}{10000}\right) = \frac{1794}{10000}$.

(2) Weighted selection value for $m_1 = \bar{s}_2 = \left(1 \otimes \frac{1710}{10000}\right) + \left(1 \otimes \frac{1591}{10000}\right) + \left(0 \otimes \frac{1794}{10000}\right) = \frac{3301}{10000}$.

(3) Weighted selection value for $m_1 = \bar{s}_3 = \left(0 \otimes \frac{1710}{10000}\right) + \left(1 \otimes \frac{1591}{10000}\right) + \left(0 \otimes \frac{1794}{10000}\right) = \frac{1591}{10000}$.

(4) Weighted selection value for $m_1 = \bar{s}_4 = \left(1 \otimes \frac{1710}{10000}\right) + \left(0 \otimes \frac{1591}{10000}\right) + \left(1 \otimes \frac{1794}{10000}\right) = \frac{3504}{10000}$.

(5) Weighted selection value for $m_1 = \bar{s}_1 = \left(0 \otimes \frac{1710}{10000}\right) + \left(1 \otimes \frac{1591}{10000}\right) + \left(1 \otimes \frac{1794}{10000}\right) = \frac{3385}{10000}$.

Hence the superimum weighted selection value = $\text{Sup}\{\bar{s}_1, \bar{s}_2, \bar{s}_3, \bar{s}_4, \bar{s}_5\} = \bar{s}_4$.

Therefore the superimum weighted selection value was exist for the selection value was $\bar{s}_4$ of m_4. Therefore the customer was chose m_4 as be the best moter cycle.

V.Conclusion

This paper was to combine the idea of a fuzzy vague parameter set and acquire another methodology for overseeing uncertainties. The meaning of the combination of a fuzzy vague parameter set is more reasonable in light of the fact that it includes uncertainty in the choice of a fuzzy vague set compared to every decision taken. A combination of fuzzy vague parameter set relations is characterized their properties are considered and decision taking is explained to delineate their applications. To extend this work, one could contemplate the topological structure for a combination of fuzzy vague parameter sets. We trust that our work upgrades the comprehension of a combination of interval-valued fuzzy vague parameter sets for future work.

Author contributions

Venkatesan J**:** Fuzzy Algebra, Functional equation, fluid dynamics,

Geetha Kannan**:** dynamics, Ring theory, fuzzy theory, Lambodharan Velappan: Fuzzy algebra, Functional equation, Ring Theory.

Conflicts of interest

The authors declare no conflicts of interest.

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