International Journal of
INTELLIGENT SYSTEMS AND APPLICATIONS IN ENGINEERING
ISSN:2147-6799 www.ijisae.org Original Research Paper

Study of the Fluid Flow in Channel of Conduction MHD Pump

Dr. Sonia Naceur

Submitted: 13/01/2024 **Revised:** 05/05/2024 **Accepted:** 12/05/2024

Abstract : This paper deals with the analysis of a coupling between stationary Maxwell's equations and the transient state Navier-Stokes equations in a conduction MHD pump .The resolution of these equations is obtained by introducing the magnetic vector potential **A** , the vorticity ξ and the stream function ψ using the finite volume method. The flux density, the electromagnetic thrust, and the variation of the velocity in the channel of the MHD pump for different positions and different conducting fluids are graphically presented.

Keywords: *Magnetohydrodynamic, Conduction pump, velocity, Maxwell's equations, Navier-Stokes, Electrode, Finite Volume Method.*

NOMENCLATURE

A	magnetic vector potential	V	velocity of the fluid
B	magnetic induction	σ	electrical conductivity
Jex	excitation current density	μ	magnetic permeability
Ji	induced eddy currents	p	pressure of the fluid
F	electromagnetic thrust	υ	kinemactic viscosity of the fluid
ξ	vorticity vector	ρ	fluid density

1. Introduction

Magnetohydrodynamics (MHD) is the study of the motion of electrically conducting fluids in the presence of magnetic fields. Effects from such interactions can be observed in liquids and gases. A number of researches have investigated the flow of an electrically conducting fluid through channels because of its important applications in MHD generators, pumps, accelerators, flowmeters and blood flow measurements. The pumping of liquid metal may use an electromagnetic device, which induces eddy currents in the metal. These induced currents and their associated magnetic field generate the Lorentz force whose effect can be actually the pumping of the liquid metal, [1-8].

The advantage of these pumps which ensure the energy transformation is the absence of moving parts.

In order to explain the basic operation principle of Magnetohydrodynamics (MHD) pump, the schematic structure of the pump is shown in figure (1), with the electrodes being on the sides and the magnetic field applied vertically. In the pump, the pumping forces are originated from the Lorentz forces induced by interaction between the applied the electrical currents and the magnetic fields, [3,5].

The basic design concept is to apply electrical currents across a channel filled with electrically conducting liquid (mercury) and magnetic fields orthogonal to the currents via electromagnetic circuit, [8,9].

Electrical Engineering Department, University Kasdi Merbah; Ouargla, Algeria.
E mail: naceursonia2@gmail.com

In the previous work, [16], we have studied the electromagnetic phenomena in a MHD conduction pump.

The purpose of this paper is to determine the velocity profile in the channel of conduction magnetohydrodynamic pump using the finite volume method**.**

2. Governing Equation

2. 1. ELECTROMAGNETIC PROBLEM

For the given basic structure shown in the figure (1) governing equations for the flow motion based upon the Lorentz forces are derived with the assumption that steady state with magnetic and electric properties of the fluid. From the basic definition for the interactions of electric charge under magnetostatic filed, the Lorentz forces can be interpreted as follows vector notation, [13,14]:

$$\vec{F} = (\vec{J}_{ind} + \vec{J}_a) \wedge \vec{B} \tag{1}$$

Where $\vec{B}$ is the magnetic flux density, $\vec{J}$ is the current density vector. And $\vec{J}$ is further defined from the Ohm's Law for electric conductor as:

$$\vec{J}_{ind} = \sigma\left(\vec{E} + \vec{V} \wedge \vec{B}\right) + \vec{J}_{ex} \tag{2}$$

The Maxwell's equations applied to the MHD pump will give rise to the following equation:

$$\vec{R}ot(\frac{1}{\mu}\vec{R}ot\vec{A}) = \vec{J}_{ex} + \vec{J}_a + \sigma(\vec{V} \wedge \vec{B}) \tag{3}$$

Where **A** is the magnetic vector potential, **σ** is the electrical conductivity, μ the magnetic permeability, V is the velocity of the fluid and **Jex** the excitation current density. For the calculation reported in the following, mercury is considered as the fluid. After developing the above equations in Cartesian coordinates:

$$\frac{1}{\mu}(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\nabla y^2}) = J_{ex} + J_a + \sigma\left(V\frac{\partial A}{\partial x}\right) \tag{4}$$

To solve this equation and to ensure the unicity of $\vec{A}$, we generally adds the condition of Coulombs jauge which is $Div\vec{A} = 0$. This assumption is naturally checked in the (2d) configuration.

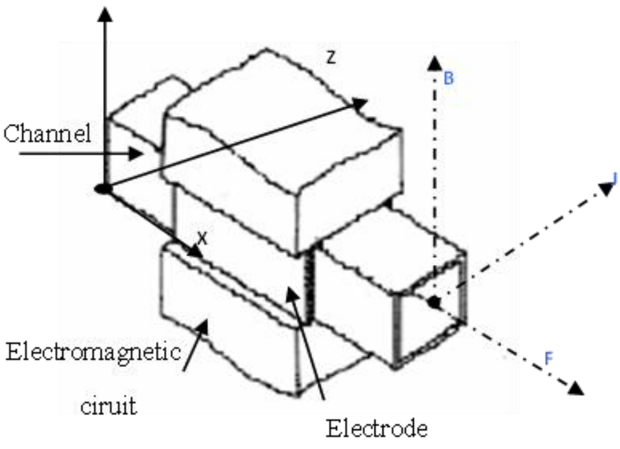

Fig.1. Scheme of a DC MHD pump,[3].

2.2. The Hydrodynamic Problems

The MHD flow of an incompressible, viscous and electrically conducting fluid in a transient state condition is governed by the Navier-Stokes equations [9]:

$$\frac{\partial\vec{V}}{\partial t} + (\nabla.\vec{V})\vec{V} = -\frac{1}{\rho}\nabla p + \nu\Delta\vec{V} + \frac{\vec{F}}{\rho} \tag{5}$$

$$div\vec{V} = 0$$

Where **p** the is the pressure of the fluid, **υ** the kinemactic viscosity of the fluid, **F** the electromagnetic thrust and **ρ** the fluid density, [12,14].

The development of the equation of the flow in Cartesian coordinates gives, [11,15]

$$\frac{\partial Vx}{\partial t}+Vx.\frac{\partial Vx}{\partial x}+Vy.\frac{\partial Vx}{\partial Vy}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 Vx}{\partial^2 x^2}+\frac{\partial^2 Vx}{\partial^2 y^2}\right]+\frac{1}{\rho}Fx$$
$$\frac{\partial Vy}{\partial t}+Vx.\frac{\partial Vy}{\partial x}+Vy.\frac{\partial Vy}{\partial Vy}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+\nu\left[\frac{\partial^2 Vy}{\partial^2 x^2}+\frac{\partial^2 Vy}{\partial^2 y^2}\right]+\frac{1}{\rho}Fy$$
$$\frac{\partial Vx}{\partial x}+\frac{\partial Vy}{\partial Vy}=0 \qquad (6)$$

The Real Difficulty Is The Calculation Of The Velocity Lies In The Unknown Pressure. To Overcome This Difficulty Is To Relax The Incompressibility Constraint In An Appropriate Way. . So, The Elimination Of Pressure From The Equations Leads To A Velocity-Stream Function The Velocity Vector Is Defined By :

$$\xi = rotV \qquad (7)$$

The stream function is given in 2D Cartesian coordinates as:

$$\frac{\partial \psi}{\partial y}=Vx; \quad \frac{\partial \psi}{\partial x}=Vy \qquad (8)$$

Where **Vx** And **Vy** The Components Of The Velocity **V**.

We Eliminate The Pressure From The Equation (12) And We Use The Two New Dependent Variables Ξ And Ψ To Obtain The Following Equation:

$$\frac{\partial \zeta}{\partial t}+Vy.\frac{\partial \zeta}{\partial y}+Vx.\frac{\partial \zeta}{\partial x}=\nu\left[\frac{\partial^2 \zeta}{\partial^2 x^2}+\frac{\partial^2 \zeta}{\partial^2 y^2}\right]+\frac{1}{\rho}\left(\frac{\partial Fx}{\partial y}+\frac{\partial Fy}{\partial y}\right) \qquad (9)$$

After substituting equation (8) into equation (9) we obtain an equation involving the new dependant variables **ξ** and **ψ** such as :

$$-\zeta = \frac{\partial^2 \psi}{\partial x^2}-\frac{\partial^2 \psi}{\partial y^2} \qquad (10)$$

2. Numerical Method

There are several methods for the determination of the electromagnetic fields and the velocity; the choice of the method depends on the type of problem, [10,11].

In our work, we thus choose the finite volume method, its principle consists on subdividing the field of study (Ω) in a number of elements. Each element contains four nodes of the grid. A finite volume surrounds each node of the grid (Fig.2), [12,13].

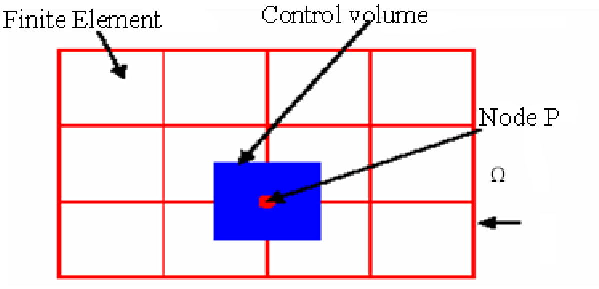

Fig.2 Grid of the domain.

In the two-dimensional case, finite volume (Fig.3) is limited by the interfaces (**E**, **W**, **N** and **S**), each principal node **P** is surrounded by four nodes: the east **E**, the west **W**, following **X**, and two following **Y**, the north **N** and the south

$$\int_w^e\int_s^n\left[\frac{1}{\mu}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}\right)\right]dx\,dy=\int_w^e\int_s^n(J_{ex}+J_a+\sigma V\frac{\partial A}{\partial x})dx\,dy \qquad (11)$$

After integration, the final algebraic equation will be:

$$a_p A_p = a_e A_e + a_w A_w + a_n A_n + a_s A_s + d_p \qquad (12)$$

With:

$$a_e = \frac{\Delta y}{\mu_e (\delta x)_e},\ a_w = \frac{\Delta y}{\mu_w (\delta x)_w},\ a_n = \frac{\Delta x}{\mu_n (\delta y)_n},\ a_s = \frac{\Delta x}{\mu_s (\delta y)_s},$$

$$a_p = a_e + a_w + a_n + a_s \,;\, d_P = (J_{ex} + J_a).\Delta x \Delta y$$

We use the same steps for the hydrodynamic problem:

$$\int_{tf}^{t}\int_{s}^{n}\int_{w}^{e}\left(\frac{\partial\zeta}{\partial t} + Vy.\frac{\partial\zeta}{\partial y} + Vx.\frac{\partial\zeta}{\partial x}\right)dxdydt = \iiint_{tf\,s\,w}^{n\,e}\left(v\left[\frac{\partial^2\zeta}{\partial^2 x^2} + \frac{\partial^2\zeta}{\partial^2 y^2}\right]\right)dxdydt$$

$$+\iiint_{tf\,s\,w}^{t\,n\,e}\left(-\frac{1}{\rho}\left(\frac{\partial Fx}{\partial y} + \frac{\partial Fy}{\partial y}\right)\right)dxdydt \qquad (13)$$

$$\int_{s}^{n}\int_{w}^{e}\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2}\right)dxdy = -\int_{s}^{n}\left(\int_{w}^{e} \zeta dxdy\right. \qquad (14)$$

The resolution of the electromagnetic and the hydrodynamic equations makes it possible to determine the magnetic potential vector, magnetic induction $(\vec{A}, \vec{B})$ the Electromagnetic force F, and the velocity in the channel of the conduction pump.

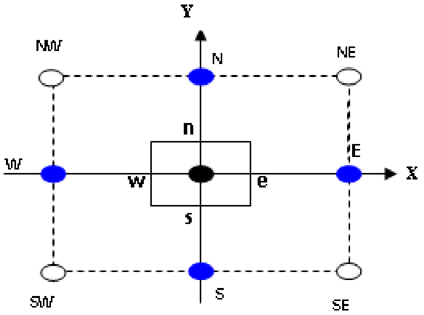

Fig.3. Description of finite volume [11].

4. Application and Results

We consider the following figure (Fig 4) which represents the transverse section of MHD pump with the following characteristics:

-The liquid in the channel is mercury with the conductivity is $\boldsymbol{\sigma}_{mercure} = 1.66*10^6$ [S/m] ;

- Current source density is **Jex**=$1.8*10^6$ [A/m²]
- Current density in the electrodes is **Ja**=$1.5*10^6$ [A/m²].

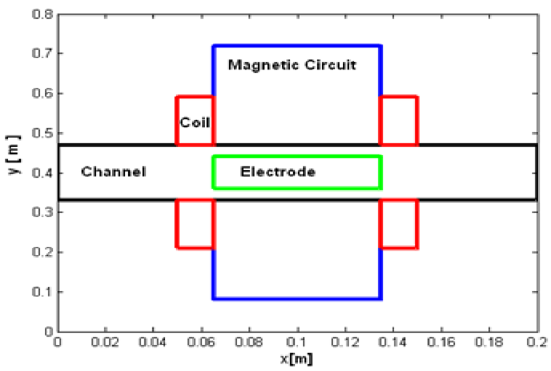

Fig 4. A conduction MHD pump configuration

The figure (5) represents the organigramme of the code used for the resolution of the electromagnetic and the hydrodynamics equations

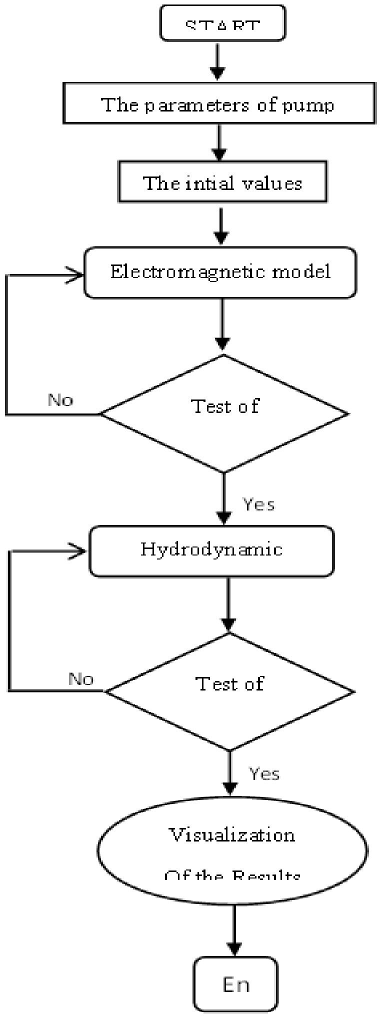

Fig 5 Computation Algorithm.

The figures (6) and (7) represent respectively the equipotential lines and the distribution of the magnetic vector potential in the MHD pump.

The figures (8) represent the magnetic induction in the channel. It is shown that, the magnetic induction reaches its maximum value at the inductor and in the medium of the channel.

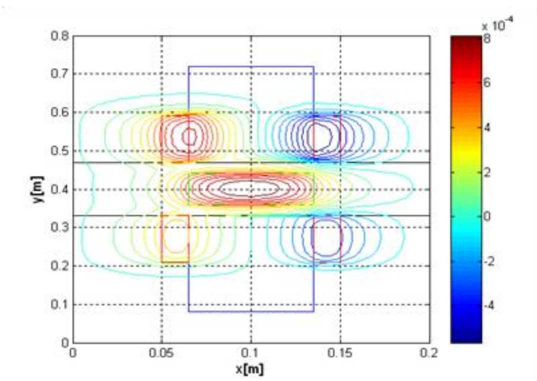

Fig.6 Equipotential lines in DC pump MHD

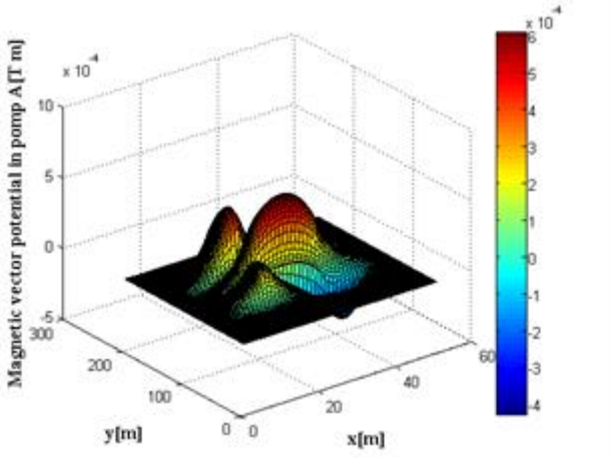

Fig 7a. Magnetic vector potential in pomp 3D

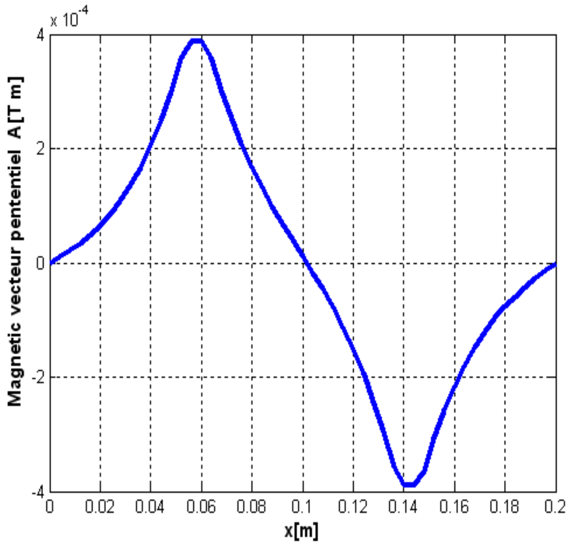

Fig 7b. Magnetic vector potential in pomp

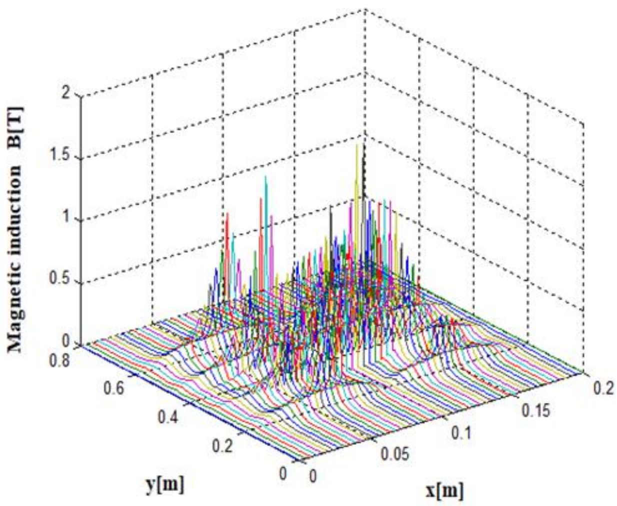

Fig.8a Magnetic induction in the MHD pump 3D

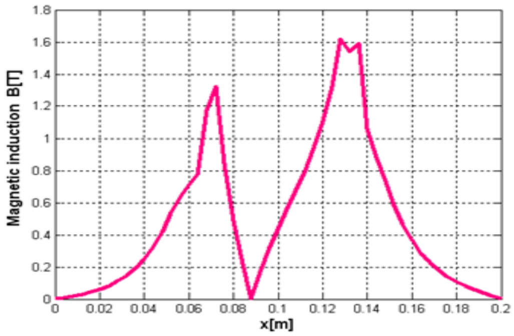

Fig.8b Magnetic induction in the MHD pump 2D

This figure (9) represents the electromagnetic force in the channel, it is note that, the maximum value in the medium of the channel of the MHD pump.

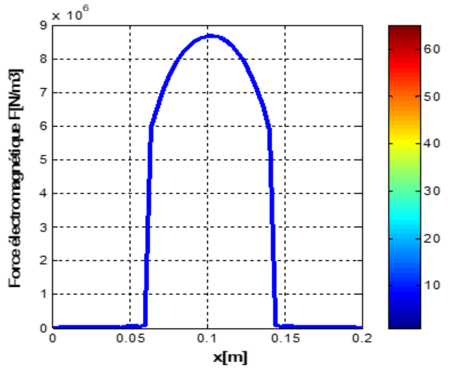

Fig 9 Electromagnetic forces in the channel

The figure (10) represents the variation of the velocity in the channel of the MHD pump for different positions; in the entry (a), in the medium (b) and the exit (c) of the channel. And different conduction fluid (mercury (red), gallium (blue), and sea water (black) It is noticed that the velocity of the fluid flow passes by a transitory mode then is stabilized like all the electric machine and the steady state is obtained approximately after ten seconds. The results obtained are almost identical qualitatively to those obtained by [6,13].

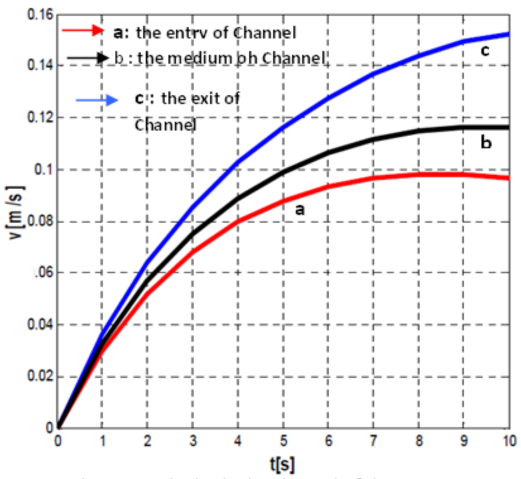

Fig 10a: Velocity in the channel of the MHD pump.

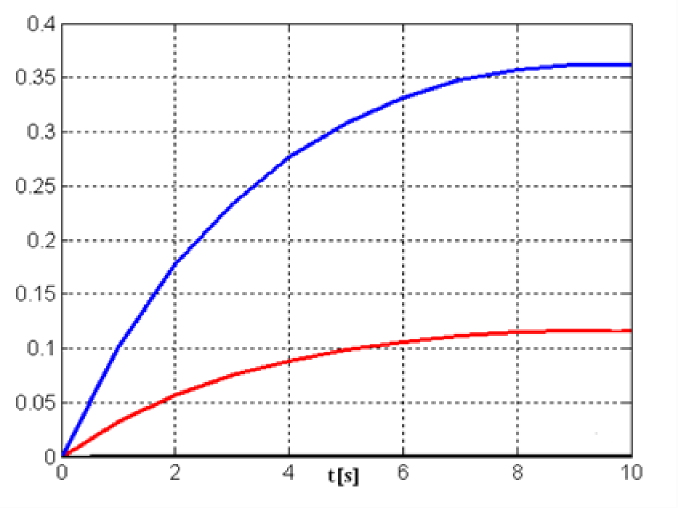

Fig 10b: Velocity in the channel of the MHD pump

5. Conclusion

In this article, we have studied the magnetohydrodynamic phenomena in 2D of pump MHD with conduction taking account of the movement of the fluid using finite volume method. Various characteristics such as the distribution of the magnetic vector potential, the magnetic induction, the electromagnetic force and the velocity are given..

In order to perform this work, it is necessary to determine the pressure and the thermal phenomena in the channel of MHD pump.

References

[1] L. Leboucher, “Optimisation Des Convertisseurs MHD à Induction : Problème Inverse En Electromagnétisme ” , Thèse de doctorat, Institut National Polytechnique De Grenoble, Grenoble, France, 1992.

[2] L. Leboucher, P. Boissonneau ‘’Chamel Shap Optimization of Electromagnetic PumpsLEGI, Institut de Mbanique de Grenoble, 38041 Grenoble cedex, FrancD. Villani

[3] L.Leboucher, P.Marty, A.Alemany,“An Inverse Method In Electromagnétism Applied To The Optimisation Of Inductors”, IEEE Transaction On Magnetics, Vol. 28, No. 5, September 1992.

[4] Andrea Cristofolini and Carlo A. Borghi’’ A Difference Method For The Solution Of The Electrodynamic Problem In A Magnetohydrodynamic Field’’, Istituto di Elettrotecnica, Universita di Bologna, Vide Risorgimento 2,40136 Bologna, Italy, IEEE transactions on magnetics, vol. 31. NO. 3, MAY 1995

[5] J.Zhong; Mingqiang Yi;Haim H.Bau; ‘’Magneto hudrodynamic (MHD) pump fabricated with ceramic tapes’’ Sesons and Actuators A 96 59-66, 2002.

[6] P.J. Wang, C.Y. Changa, M.L. Changb, ’’Simulation of two-dimensional fully developed laminar flow for a magneto-hydrodynamic (MHD) pump, ELSVIER, Biosensors and Bioelectronics 20, pp 115-121, 2004.

[7] D. Convert ,’’Propulsion Magnétohydrodynamique en eau de mer ‘’ ’Thèse de Doctorat, Université de Grenoble, 1995.

[8] Chia-Yuan Chang ‘’ Analysis of MES-SCALE heat exchangers with magneto-hydrodynamic pumps; National Tsing hua University June 2004.

[9] P. Boissonneau, “Propulsion MHD En Eau De Mer : Etude des Couplages Hydrodynamique-Electrochimie-Electromagnétisme”, Institut de mécanique, Université Joseph Fourier Grenoble, 1997.

[10] F. Z. Kadid, “Contribution A L’étude Des Convertisseurs MHD A Induction ”, Thèse de doctorat, Institut de l’électrotechnique, Université de Batna, 2004.

[11] S.V.Patankar, “Numerical Heat Transfer Fluide Flow”, Hemisphere.

[12] Majid Ghassemi , Hojattoallah Rezaeinezhad ,and Azadeh Shahidian: Analytical Analysis of Flow in a Magnetohydrodynamic Pump (MHD); 978-1-4244-1833-6/08/$25.00 © 2008 IEEE

[13] MajidGhassemi ,AlirezaGhassemi and Masood Ziabasharhagh: ThermalStressAnalysisoftheRailsandtheArmature of an Electromagnetic Launcher; 2009 IEEE

[14] BENNECIB NEDJOUA ‘’ Contribution a l’etude d’une machine Mhd a conduction en vue de son exploitation sur un reseau électrique’’ these de doctrat 2010.

[15] Fatima Zohra Kadid — Rachid Abdessemed — Sa¨ıd Drid: Study Of The Fluid Flowin A Mhd Pump By Couplin Finite Element–Finite Volume Computation, journal of electrical engineering, vol. 55, no. 11-12, 2004, 301–305

[16] S.Naceur, F. Z. Kadid, R. Abdessemed, ‘’Etude électromagnétique d’une pompe magnétohydrodynamique (MHD) à conduction’’ 05éme Conférence Nationale sur le Génie Electrique 16-17 Avril 2007, Ecole Militaire Polytechnique EMP, Bordj el Bahri –Algérie

[17] S.Naceur, F. Z. Kadid, R. Abdessemed,‘’ Influence of the electrode length on the performances of a conduction MHD Pump’’ Proceedings ICEEDT’07, International conference on electrical engineering design and technologies ICEEDT’07, 4- 6 Novembre 2007 Hammamet, Tunisia.