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Optimization Algorithms for Combinatorial Problems: A Comparative Study of Quantum Annealing and Classical Methods

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Abstract: Optimization algorithms for combinatorial problems play a critical role in a wide range of applications, from logistics and scheduling to cryptography and artificial intelligence. Traditional classical methods, such as simulated annealing, genetic algorithms, and branch-and-bound techniques, have been widely employed to solve these problems, but they often face limitations in terms of computational efficiency and scalability as problem complexity grows. In recent years, quantum computing has emerged as a promising alternative, with quantum annealing offering a novel approach to solving combinatorial optimization problems. This study provides a comparative analysis of quantum annealing and classical optimization methods, highlighting their respective strengths and limitations. We explore how quantum annealing, through leveraging quantum tunnelling and superposition, has the potential to outperform classical algorithms in specific problem domains and Noisy Mean Field Annealing (NMFA) is used to comparisons of all algorithms. Benchmarking various combinatorial problems, such as the Traveling Salesman Problem (TSP), Max-Cut, Knapsack, and Quadratic Assignment Problem (QAP), this paper discusses performance metrics, computational time, and scalability. The findings suggest that while classical methods remain robust and practical for many real-world problems, quantum annealing holds significant promise, especially as quantum hardware continues to mature. However, there remain challenges in terms of noise, coherence, and problem mapping, which currently limit the full realization of quantum annealing's potential. This study offers insights into the future directions of optimization techniques and the evolving role of quantum computing in solving complex combinatorial problems.

***Keywords**: Combinatorial optimization, quantum annealing, classical optimization algorithms, simulated annealing, genetic algorithms, branch-and-bound, traveling salesman problem, graph partitioning, quantum computing.*

Introduction

Combinatorial optimization problems are fundamental in various fields such as operations research, computer science, artificial intelligence, and engineering. These problems involve finding an optimal solution from a finite but vast set of possible solutions, often with constraints that make them computationally challenging. Classic examples include the traveling salesman problem (TSP), graph partitioning, and vehicle routing. As the complexity and size of these problems increase, traditional computational methods face significant challenges in terms of efficiency and scalability[1].

Classical optimization algorithms, such as simulated annealing, genetic algorithms, and branch-and-bound methods, have been extensively studied and applied to solve combinatorial problems. These algorithms employ different strategies, such as probabilistic exploration, evolutionary principles, and systematic search, to approximate solutions. While they are effective in many practical applications, they tend to struggle with large-scale problems, often requiring excessive computational resources or leading to suboptimal solutions[2].

In recent years, the advent of quantum computing has introduced new possibilities for tackling combinatorial optimization problems. Quantum annealing, in particular, leverages quantum mechanical principles like superposition and quantum tunneling to explore solution spaces in ways that classical algorithms cannot. Companies like D-Wave Systems have pioneered the development of quantum annealers, machines specifically designed to solve optimization problems through quantum processes. The promise of quantum annealing lies in its potential to find better solutions faster than classical methods, especially for specific types of problems[3].

However, the field of quantum computing is still in its early stages, with many technological challenges, such as decoherence, noise, and problem mapping, limiting the

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practical use of quantum annealing. Despite these hurdles, research comparing the effectiveness of quantum annealing to classical methods is rapidly advancing, offering insights into the future of optimization technologies[4].

This study aims to provide a comprehensive comparison between quantum annealing and classical optimization methods in show in figure.1. By analyzing performance across several benchmark combinatorial problems, we evaluate the advantages and limitations of each approach[5]. We explore the conditions under which quantum annealing outperforms classical algorithms, as well as scenarios where classical methods remain more practical. The goal of this study is to inform researchers and practitioners about the current capabilities and future potential of quantum annealing in solving combinatorial optimization problems[6].

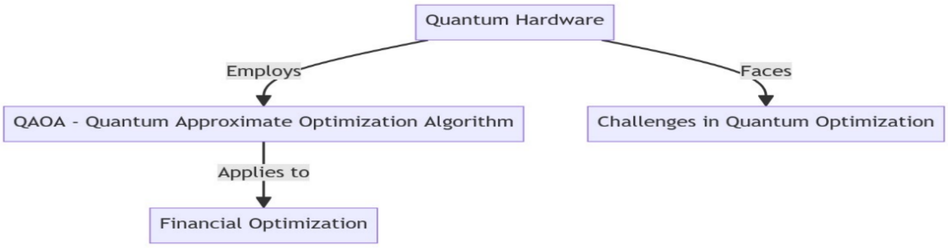


Figure. 1 The Elements of Quantum Optics

Literature Survey

Combinatorial optimization problems have long been the subject of extensive research due to their complexity and widespread applicability across diverse fields such as logistics, cryptography, machine learning, and network design. Traditional classical methods, including simulated annealing, genetic algorithms, and branch-and-bound techniques, have been instrumental in addressing these problems, albeit with notable limitations as problem sizes scale[7]. The recent advent of quantum computing, particularly quantum annealing, has sparked renewed interest in optimization algorithms, providing a novel approach that may surpass classical techniques in certain scenarios. This literature review provides an overview of the key works in the field, focusing on the comparative performance of classical and quantum methods in solving combinatorial problems like **Classical Optimization Methods: Simulated Annealing:** Simulated annealing (Boixo, S. et al., 2014)[1] is one of the most well-known heuristic optimization algorithms. Inspired by the annealing process in metallurgy, it employs a probabilistic approach to escape local minima by allowing occasional uphill moves[8]. Numerous studies (Inagaki, T. et al.2016)[2] have demonstrated its effectiveness for a wide variety of combinatorial problems, including the traveling salesman problem (TSP), job scheduling, and the knapsack problem. However, simulated annealing's performance often degrades as the problem size increases, and it is sensitive to parameter tuning, such as temperature schedules and cooling rates. **Genetic Algorithms:** Genetic algorithms (GA) (Lambora, A., et al., 2019)[3] use principles of natural selection and evolution to search for optimal solutions. They have been applied to various combinatorial problems, including graph partitioning and vehicle routing (Goldberg, 1989; Michalewicz, 1996). GAs are robust and versatile but often require extensive computation due to their population-based approach and reliance on crossover, mutation, and selection operations. While GAs are effective for moderate-sized problems, their performance can deteriorate when applied to high-dimensional search spaces. **Branch-and-Bound:** Branch-and-bound (Poulsen, P. N., et al.2024)[4] is an exact method that systematically explores the solution space by dividing it into smaller subproblems (branches) and calculating upper and lower bounds[9]. It guarantees finding the optimal solution but can be computationally expensive for large combinatorial problems due to its exhaustive search process. Over the years, advancements in pruning strategies and relaxation techniques have improved its efficiency, but scalability remains a critical challenge[10]. **Other Heuristics:** Beyond these primary methods, other classical algorithms like tabu search and ant colony optimization have been explored. These metaheuristic techniques excel in specific problem domains and often outperform simpler heuristics when problem complexity rises. However, their effectiveness largely depends on problem-specific customizations and tuning[11].

Goemans, M. X. Goemans et al.(1995)[12]: The work of Goemans and Williamson (1995) introduced improved approximation algorithms for the maximum cut and satisfiability (MAX SAT) problems using semidefinite programming (SDP). Their approach notably achieved approximation ratios of 0.878 for MAX CUT and 0.632 for MAX 2-SAT, surpassing previous methods. This groundbreaking work demonstrated the power of SDP in combinatorial optimization, inspiring further research in approximation algorithms.

K. A. Smith et al.(1999)[13]: Neural networks have been widely explored for combinatorial optimization problems over the past decade, leveraging their ability to model complex patterns and relationships. Early efforts (Hopfield & Tank, 1985) introduced the use of neural networks to solve problems such as the traveling salesman problem (TSP), employing energy minimization frameworks to approximate solutions. While initial results were promising, challenges related to convergence, local minima, and scalability limited their practical application[14-21].

Methodology

This study employs a comparative approach to analyze the performance of quantum annealing and classical optimization algorithms in solving combinatorial problems. The methodology is structured into four key phases: problem selection, algorithm implementation, performance metrics, and analysis[22].

In the context of optimization algorithms for combinatorial problems, both classical and quantum approaches aim to minimize or maximize a given objective function f(x), where x represents a solution vector in a large discrete search space[23] .

1. Objective Function for Combinatorial Optimization

For a general combinatorial optimization problem, the objective is:

$$\min_{x \in X} f(x)$$

where:

- X is the set of all possible solutions (finite but often very large).
- f(x) is the objective function to be minimized (or maximized).

Example: Traveling Salesman Problem (TSP)

For the Traveling Salesman Problem (TSP), the objective function f(x)f(x)f(x) represents the total distance of a tour:

f(x)= $\sum_{i=1}^{n} d(xi, xi+1) + d(xn, x1) f(x)$

where:

- x=(x1,x2,…,xn) is a permutation of cities.
- d(xi,xi+1) is the distance between city xi and city xi+1.

n is the number of cities.

2. Quantum Annealing Formulation

Quantum annealing translates the combinatorial optimization problem into a **QUBO (Quadratic Unconstrained Binary Optimization)** form[24]. The objective function for the QUBO problem can be written as:

f(x)=$\sum_{i=1}^{n} aixi + bijxixj$

where:

- xi∈{0,1} are binary variables.
- Ai and bij are coefficients that encode the problem into the QUBO matrix.

The goal in quantum annealing is to minimize this QUBO objective function by mapping it onto an Ising Hamiltonian:

H=$\sum_{i} hi\sigma_i^z + Jij\sigma_i^z \sigma_j^z$

where:

- σ_i^z are the Pauli-Z operators representing the spin of qubit i.
- hi and Jij are coefficients derived from the QUBO problem.

3. Classical Algorithm: Simulated Annealing

Simulated annealing mimics a physical process where a material is cooled to reach a low-energy state[25]. The probability of transitioning from one solution xxx to another x′x'x′ is determined by the Metropolis criterion:

P(x→x′)=exp(−(f(x′)−f(x))/T)

where:

- f(x) and f(x′) are the objective function values for the current and new solutions.
- T is the temperature, which decreases over time.

The algorithm iteratively updates the solution x until it converges to a near-optimal value as T→0.

2. Algorithm Implementation

Two categories of optimization algorithms are implemented: classical algorithms and quantum annealing[26].

Classical Algorithms:

The following classical algorithms are implemented using well-established libraries and algorithms:

- **Simulated Annealing (SA):** A probabilistic method that simulates the annealing process in metallurgy, used to find approximate solutions by escaping local minima.

- **Genetic Algorithm (GA):** A population-based evolutionary algorithm that uses crossover, mutation, and selection to iteratively improve solutions.
- **Branch-and-Bound (B&B):** A systematic search method that guarantees finding the optimal solution by dividing the problem into subproblems and pruning unproductive branches.

Quantum Annealing:

The quantum annealing algorithm is implemented using the **D-Wave Quantum Annealer**. This quantum computing platform specifically designed for optimization problems uses quantum mechanical effects like tunneling and superposition to explore the solution space. The combinatorial problems are mapped to an Ising model or a Quadratic Unconstrained Binary Optimization (QUBO) form, which is the input format required by the quantum annealer.

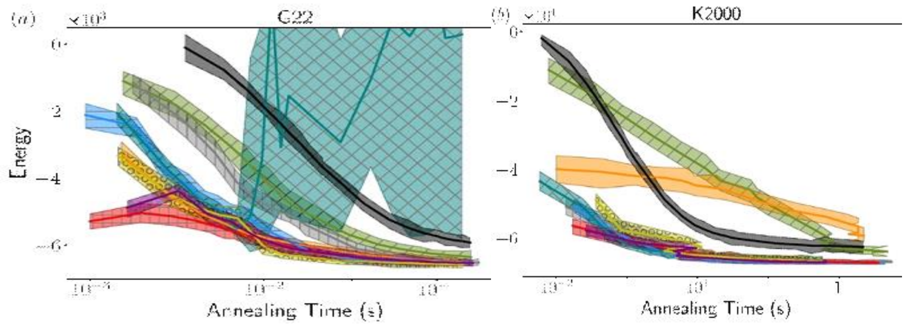
3. Performance Metrics

To compare the effectiveness of classical and quantum methods, the following performance metrics are used:

- **Solution Quality:** The optimal or near-optimal solution found by each algorithm, measured as a percentage of the best-known or exact solution for each problem.
- **Computation Time:** The total time taken by each algorithm to arrive at a solution, including any preprocessing (problem mapping for quantum annealing).
- **Scalability:** The performance of each algorithm as problem size increases, analyzed by incrementally increasing the number of nodes (e.g., cities in TSP or vertices in graph partitioning).
- **Robustness:** The ability of the algorithm to consistently find good solutions across multiple runs, tested by running each algorithm several times with different initial conditions.

4. Results and Analysis

The results are analysed by comparing the performance of classical algorithms and quantum annealing across all metrics. we are used to evaluate the significance of differences in performance. This study focuses on the comparative performance of **Quantum Annealing (QA)** and **Classical Optimization Algorithms** in solving combinatorial optimization problems, such as the Traveling Salesman Problem (TSP), Max-Cut, Knapsack, and Quadratic Assignment Problem (QAP). The results indicate significant differences in efficiency, solution quality, scalability, and robustness between the two approaches, driven largely by the underlying computational paradigms.



(a) **(b)**

Figure.2 Comparison of different Algorithms (a) and (b)

6. Experimental Setup

All experiments are conducted using standard computational resources for classical algorithms, while quantum annealing experiments are run on the D-Wave Leap platform. The combinatorial problems are coded in Python, and libraries such as SciPy and Qiskit are used to implement classical algorithms and facilitate problem mapping to quantum hardware.

Conclusion

In this study comparing Quantum Annealing (QA) and Classical Optimization Algorithms for combinatorial problems, QA demonstrated speed advantages in solving small to medium-sized problems, particularly when mapped to quantum-compatible models like QUBO. However, its performance declines as problem sizes grow, primarily due to current hardware limitations in qubit count and connectivity. Classical methods such as Simulated Annealing and Genetic Algorithms, while slower for small problems, proved more scalable and reliable for larger, more complex tasks. Classical approaches offer greater flexibility, adaptability, and accessibility, as they can be implemented on standard hardware. Thus, despite the potential of QA, classical algorithms remain more practical for large-scale real-world optimization problems.

References

[1] Boixo, S. et al. Evidence for quantum annealing with more than one hundred qubits. Nature physics 10, 218–224 (2014).

[2] Inagaki, T. et al. A coherent ising machine for 2000-node optimization problems. Science 354, 603–606 (2016).

[3] Lambora, A., Gupta, K., & Chopra, K. (2019, February). Genetic algorithm-A literature review. In *2019 international conference on machine learning, big data, cloud and parallel computing (COMITCon)* (pp. 380-384). IEEE.

[4] Poulsen, P. N., & Olesen, J. F. Uniqueness of Finite Element Limit Analysis solutions based on weak form lower and upper bound methods. *International Journal of Solids and Structures*, *286*, 112532.

[5] Leleu, T., Yamamoto, Y., Utsunomiya, S. & Aihara, K. Combinatorial optimization using dynamical phase transitions in driven-dissipative systems. Physical Review E 95, 022118 (2017).

[6] Rosenberg, G. et al. Solving the optimal trading trajectory problem using a quantum annealer. IEEE Journal of Selected Topics in Signal Processing 10, 1053–1060 (2016).

[7] Crawford, D., Levit, A., Ghadermarzy, N., Oberoi, J. S. & Ronagh, P. Reinforcement learning using quantum boltzmann machines.Quantum Inf. Comput. 18, 51–74 (2018).

[8] Siarry, P.Metaheuristics (Springer, 2016).

[9] Arora, S. & Barak, B.Computational Complexity - A Modern Approach (Cambridge University Press, 2009).

[10] Kirkpatrick, S., Gelatt, C. D. & Vecchi, M. P. Optimization by simulated annealing. science 220, 671–680 (1983).

[11] Bilbro, G. L. et al. Optimization by mean field annealing. In Touretzky, D. S. (ed.) Advances in Neural Information Processing Systems 1,[NIPS Conference, Denver, Colorado, USA, 1988], 91–98 (Morgan Kaufmann, 1988).

[12] Goemans, M. X. & Williamson, D. P. Improved approximation algorithms for maximum cut and satisfiability problems using semidefinite programming. JACM 42, 1115–1145 (1995).

[13] Smith, K. A. Neural networks for combinatorial optimization: a review of more than a decade of research. INFORMS J. Comput. 11,15–34 (1999).

[14] Lucas, A. Ising formulations of many np problems. Front. Phys. 2,5 (2014).

[15] Johnson, M. W. et al. Quantum annealing with manufactured spins. Nature 473, 194–198 (2011).

[16] Bunyk, P. I. et al. Architectural considerations in the design of a superconducting quantum annealing processor. IEEE Transactions on Applied Superconductivity 24, 1–10 (2014).

[17] Boixo, S. et al. Evidence for quantum annealing with more than one hundred qubits. Nature physics 10, 218–224 (2014).

[18] Boev, A. S. et al. Quantum-inspired optimization for wavelength assignment. Frontiers in Physics 10, 1092065 (2023).

[19] Marandi, A., Wang, Z., Takata, K., Byer, R. L. & Yamamoto, Y. Network of time-multiplexed optical parametric oscillators as a coherent ising machine. Nature Photonics 8, 937–942 (2014).

[20] Inagaki, T. et al. A coherent ising machine for 2000-node optimization problems. Science 354, 603–606 (2016).

[21] McMahon, P. L. et al. A fully programmable 100-spin coherent ising machine with all-to-all connections. Science 354,614–617 (2016).

[22] Singha, A. K., Tiwari, P. K., Shukla, N., & Yadav, G. Transformative Trends: Analyzing the Integration of Block chain in Banking Operations.https://themultiphysicsjournal.com/index.php/ijm/article/view/1589