

Effects of Radiation and Chemical Reaction on Magnetohydrodynamic Elastico-Viscous Fluid Flow Past an Impulsively Started Infinite Vertical Plate with Hall Effect

Krishna Kumari¹ Ayaz Ahmad²

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Abstract: An unstable Magnetohydrodynamic free convection flow of elasto-viscous fluid past an infinite vertical plate is shown, taking into consideration the homogeneous chemical reaction of first order and radiation. The temperature and concentration are expected to be fluctuate with time. Assuming constant suction at the plate, solutions have been obtained for velocity, temperature and concentration distributions by using method of separation of variables and presented graphically, for various values of the elastic parameter (α), Schmidt number (Sc), Magnetic parameter (M) and Hall parameter (m), Radiation Parameter (F) and chemical reaction parameter (Kc).

Keywords: Hall effect, elasto-viscous, Heat-mass transfer, Radiation, Chemical Reaction.

Introduction

The phenomena of heat and mass transfer has been the target of much research due to its applications in science and technology. In nature and industrial applications numerous transport mechanisms exist where the heat and mass transfer takes occur concurrently as a result of combined effects of thermal diffusion and diffusion of chemical species. Furthermore, heat mass and transfer is also found in chemical processes industries such as food processing and polymer synthesis. An substantial contribution on heat and mass transfer fluxes has been made by Khair and Bejan [1]. Lin and Wu [2] have analyzed the problem of simultaneous heat and mass transfer with complete range of buoyancy ratio for most practical chemical species in dilute and watery solutions. Muthukumarswamy et al. [3] evaluated the heat and mass transfer impacts on flow via an impulsively initiated infinite vertical plate. The answer was derived using the Laplace transform approach, and the impacts of Grashof number, Prandtl number and Schmidt number were examined. It is generally accepted that a number of industrial fluids such as molten plastics, polymeric liquids, food stuffs or slurries showcase non-Newtonian fluid behaviour. Therefore, heat and mass convey in non-Newtonian fluid is of practical importance. Das and Biswal [4] examined the mass transport on visco-elastic fluid. Wang [5] investigated mixed convection from a vertical plate to non-Newtonian fluid with homogeneous surface heat flux. Sarpakaya [6] has presented many probable uses of non-Newtonian fluids in various

sectors. The flow of visco-elastic fluids in the presence of magnetic field have been explored by Singh and Singh [7] and Sherief and Ezzat [8]. In an ionized gas where the density is low and (or) the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions; also a current is induced in a direction normal to both electric and magnetic field. This phenomenon is well known in the literature and is called the Hall effect. Sato [9] and Sherman and Satton [10] were the first authors who investigated the hydromagnetic flow of ionized gas between two parallel plates taking Hall effect into account. The effect of Hall current for MHD free convection flow along a vertical surface and in the presence of transverse magnetic field with or without mass transfer have been studied by number of authors; Pop [11], Raptis and Ram [12], Hossain and Rashid [13], Hossain and Mohammad [14], Pop and Watanabe [15], Acharya et. al. [16, 17], Abodeldhab and Elbarabary [18] and Asghar et. al. [19]. Chaudhary, and Jha, [20] discussed heat mass transfer with hall effect on flow past a vertical plate. The study of chemical reaction plays an important role in processes such as dyeing, distribution temperature and moisture over agricultural fields and groves of fruits, energy transfer in a wet cooling tower and flow in desert cooler. Effects of chemical reaction on polar fluid flow past a vertical plate in porous medium has been studied by Chaudhary and Jha [21] and Patil, Kulkarni [22]. Ibrahim and Makinde, [23] studied radiation effect on chemically MHD boundary layer flow of heat and mass transfer through a porous vertical flat plate. MHD free convective flow of visco elastic fluid through porous media in

¹Department of Mathematics, L.N. Mithila University, Darbhanga, Bihar

²Department of Mathematics, L.N. Mithila University, Darbhanga, Bihar
Email-karnkrishna3@gmail.com

presence of radiation and chemical reaction have been studied by Chaudhury and Das [24] and Popoola et. al [25]. Suneetha, Reddy [26] investigated effects of radiation and chemical reaction on MHD mixed convective flow with Ohmic heating and viscous dissipation

In the present analysis, it is proposed to study the effects of chemical reaction and radiation on the flow of elastico-viscous fluid past an impulsively started infinite vertical plate taking Hall effect into the account. Closed form solutions have been obtained for the velocity, temperature, concentration distribution and are shown graphically.

Mathematical Formulation

The constitutive equations for the rheological equation of state for an elastico-viscous fluid (Walter's liquid B') [27] are

$$p_{ik} = -p g_{ik} + p'_{ik} \quad (1)$$

$$p'_{ik} = 2 \int_{-\infty}^t \psi(t-t') e_{ik}^{(1)}(t') dt' \quad (2)$$

in which

$$\psi(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{-(t-t')\tau} d\tau \quad (3)$$

$N(\tau)$ is the distribution function of relaxation times. In the above equations p_{ik} is the stress tensor, p an arbitrary isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x_i and $e_{ik}^{(1)}$, the rate of strain tensor. It

was shown by Walter's [24] that equation (2) can be put in the following generalized form which is valid for all types of motion and stress

$$p^{ik}(x, t) = 2 \int_{-\infty}^t \psi(t-t') \frac{\partial x^i}{\partial x'^m} \frac{\partial x^k}{\partial x'^r} e^{(1)mr}(x' t') dt' \quad (4)$$

where x^i is the position at time t of the element that is instantaneously at the point x^i at time " t ". The fluid with equation of state (1) and (4) has been designated as liquid B'. In the case of liquids with short memories, i.e. short relaxation times, the above equation of state can be written in the following simplified form

$$p^{ik}(x, t) = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\partial e^{(1)ik}}{\partial t}, \quad (5)$$

in which $\eta_0 = \int_0^{\infty} N(\tau) d\tau$ is limiting viscosity

at small rates of shear, $k_0 = \int_0^{\infty} \tau N(\tau) d\tau$ and $\frac{\partial}{\partial t}$ denotes the convected time derivative.

We consider the unsteady flow of a viscous incompressible and electrically conducting elastico-viscous fluid with oscillating temperature and concentration. We consider the flow along x-axis which is taken to be along the plate and y-axis is taken normal to it. The plate starts moving in its own plane with velocity U_0 (a constant velocity). A uniform magnetic field is applied normal to the plate with constant suction as shown in figure 1. The equations governing the flow of fluid together with Maxwell's electromagnetic equations are as follows

Equation of Continuity

$$\nabla \cdot \mathbf{V} = 0 \quad (6)$$

Momentum Equation

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho} \nabla P + \nabla \cdot \mathbf{p}_{ij} + \mathbf{g} \beta (T - T_{\infty}) + \mathbf{g} \beta^* (C - C_{\infty}) + \frac{1}{\rho} (\mathbf{J} \times \mathbf{B}) \quad (7)$$

Maxwell Equations

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_m \mathbf{J}, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (8)$$

Ohm's Law

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B}) - \frac{\sigma}{en_e} (\mathbf{J} \times \mathbf{B} - \nabla p_e) \quad (9)$$

where $\mathbf{V} = (u, v, w)$ is the velocity field, P is the pressure field, \mathbf{g} is acceleration due to gravity, β the volumetric coefficient of the thermal expansion, β^* the volumetric coefficient of expansion with concentration, ρ the density of the fluid, \mathbf{J} is the current density, \mathbf{B} is the magnetic field, \mathbf{E} is the electric field, μ_m is the magnetic

permeability, p_{ij} is stress tensor. The effect of Hall current induces a force which causes cross flow in the z direction. Therefore the flow becomes three dimensional. It is assumed that there is no applied or polarization voltage so that $E = 0$ and the induced magnetic field is negligible so that the total magnetic field $B = (0, B_0, 0)$ where B_0 is the applied magnetic field parallel to y-axis. This assumption is justified when the magnetic Reynolds number (The ratio of the moduli of the convection term and diffusive term. This number is non-dimensional and strictly analogous in the properties and uses to the Reynolds number) is very small.

The generalized Ohm's law including Hall current is given in the form [Ref. [13] and [23].

$$\mathbf{J} + \frac{\omega_e \tau_e}{B_0} (\mathbf{J} \times \mathbf{B}) = \sigma (\mathbf{V} \times \mathbf{B} + \frac{1}{en_e} \nabla p_e) \quad (10)$$

where ω_e is the electron frequency, τ_e is the electron collision time, σ is the electrical conductivity, e is the electron charge, p_e is the electron pressure and n_e is the number density of electron. For weakly ionized gases the thermoelectric pressure and ion slip are considered negligible. Then equation (10) reduces to

$$J_x = \frac{\sigma B_0}{1+m^2} (\mu u - w) \quad (11)$$

$$J_z = \frac{\sigma B_0}{1+m^2} (u + mw) \quad (12)$$

where u and w are the x and z component of the velocity vector respectively and m is the Hall parameter defined by $m = \omega_e \tau_e$. Under this condition the Boussinesq approximation equations governing the flows are as follows

Equation of Continuity

$$\frac{\partial v}{\partial y} = 0 \quad (13)$$

$\Rightarrow v = -v_0$ where v_0 is constant suction velocity.

Momentum equation

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - k_0 \frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\sigma \beta_0^2 (u+m\omega)}{\rho(1+m^2)} + g\beta(T - T_\infty) + g\beta^*(c - c_\infty) \quad (14)$$

$$\frac{\partial \omega}{\partial t} + v \frac{\partial \omega}{\partial y} = \nu \frac{\partial^2 \omega}{\partial y^2} - k_0 \frac{\partial^3 \omega}{\partial y^2 \partial t} - \frac{\sigma \beta_0^2 (\omega - mu)}{\rho(1+m^2)} \quad (15)$$

Energy equation

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (16)$$

Concentration equation

$$\frac{\partial c}{\partial t} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} - k_1 (c - c_\infty) \quad (17)$$

Where ρ is density of the fluid, ν is the kinematic viscosity, k_0 is the elastic parameter, κ is the thermal conductivity, T is the temperature, C is concentration, C_p is the specific heat of the fluid. D is the chemical molecular diffusivity and g is the acceleration due to gravity, q_r is the radiative heat flux in the y direction, k_1 is the chemical reaction parameter.

In equation (4) the terms due to viscous dissipation are neglected and in equation (5) the term due to chemical reaction is assumed to be absent.

The initial boundary conditions are:

$$t \leq 0, u(y,t) = \omega(y,t) = 0, T = 0, C = 0 \text{ for all } y.$$

$$t \geq 0$$

$$u(0,t) = U_0, \omega(0,t) = 0, T = T_\infty + e^{i\omega t} (T_\omega - T_\infty)$$

$$C(0,t) = C_\infty + e^{i\omega t} (C_\omega - C_\infty), \text{ at } y=0$$

$$u(\infty, t) = \omega(\infty, t), T(\infty, t) = C(\infty, t) = 0 \text{ as } y \rightarrow \infty \quad (18)$$

Where ω is the frequency of oscillation and subscript ω and ∞ denotes the physical quantities at plate and in the free stream respectively.

Introducing Non-dimensional parameters

$$\eta = \frac{v_0 y}{\vartheta}, \quad \bar{t} = \frac{v_0^2 t}{4\vartheta}, \quad \bar{u} = \frac{u}{u_0}, \bar{\omega} = \frac{\omega}{u_0}, \quad \bar{\theta} = \frac{T - T_\infty}{T_\omega - T_\infty}, \bar{C} = \frac{C - C_\infty}{C_\omega - C_\infty} \quad (19)$$

$$G = \frac{4g\beta\vartheta(T_\omega - T_\infty)}{v_0^2 U_0} \quad (\text{Grashof number})$$

$$G_c = \frac{4g\beta^*\vartheta(C_\omega - C_\infty)}{v_0^2 U_0} \quad (\text{Modified Grashof number})$$

$$M = \frac{4\beta_0^2 \sigma \vartheta}{\rho v_0^2 U_0} \quad (\text{Hartman number})$$

$$P_r = \frac{\vartheta \rho C_p}{\kappa} \quad (\text{Prandtl number})$$

$$\alpha = \frac{k_0 v_0^2}{\vartheta^2} \quad (\text{Elastic parameter})$$

$$S_c = \frac{\vartheta}{D} \quad (\text{Schmidt number})$$

$$\frac{\partial q_r}{\partial y} = \frac{-16\sigma^* \partial T}{3K' \partial y}$$

$$Q = \frac{K_1}{V_0^2}$$

$$\Omega = \frac{4\vartheta\omega}{V_0^2} \quad (\text{non-dimensional frequency of oscillation})$$

Substituting equation (19) in (14)-(17) and (18) and dropping the bars, we get

$$\frac{\partial u}{\partial t} - 4 \frac{\partial u}{\partial \eta} = 4 \frac{\partial^2 u}{\partial \eta^2} - \alpha \frac{\partial^3 u}{\partial \eta^2 \partial t} - \frac{M(m\omega + u)}{1+m^2} + G\theta + G_c C \quad (20)$$

$$\frac{\partial \omega}{\partial t} - 4 \frac{\partial \omega}{\partial \eta} = 4 \frac{\partial^2 \omega}{\partial \eta^2} - \alpha \frac{\partial^3 \omega}{\partial \eta^2 \partial t} - \frac{M(\omega - mu)}{1+m^2} \quad (21)$$

$$\frac{\partial \theta}{\partial t} - 4 \frac{\partial \theta}{\partial \eta} = \frac{4}{P_r} \frac{\partial^2 \theta}{\partial \eta^2} + 4R \frac{\partial \theta}{\partial \eta} \quad (22)$$

$$\frac{\partial C}{\partial t} - 4 \frac{\partial C}{\partial \eta} = \frac{4}{S_c} \frac{\partial^2 C}{\partial \eta^2} - 4QC \quad (23)$$

And the boundary conditions for equation (20)-(23) are

$$t \leq 0, u(\eta, t) = w(\eta, t) = \theta(\eta, t) = C(\eta, t) = 0 \quad \forall \eta$$

$$t \geq 0 \begin{cases} u(0, t) = 1, w(0, t) = 0, \theta(0, t) = e^{i\Omega t}, C(0, t) = e^{i\Omega t} \\ u(\infty, t) = w(\infty, t) = 0, \theta(\infty, t) = C(\infty, t) = 0 \end{cases} \quad \text{as } \eta \rightarrow \infty \quad (24)$$

Solutions

The equations (20) and (21) can be combined using the complex variable

$$\psi = u + i\omega \quad (25)$$

Equations (20)-(21) give

$$\frac{\partial^2 \psi}{\partial \eta^2} - \alpha \frac{\partial^3 \psi}{\partial \eta^2 \partial t} - \frac{1}{4} \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial \eta} - \frac{M(1+im)}{4(1+m^2)} \psi = \frac{-G\theta}{4} - \frac{G_c C}{4} \quad (26)$$

Using eq (25), we get boundary conditions as

$$\psi(0, t) = 1, \psi(\infty, t) = 0, C(0, t) = e^{i\Omega t}$$

$$\theta(0, t) = e^{i\Omega t}, \theta(\infty, t) = 0, C(\infty, t) = 0 \quad (27)$$

Putting $\theta(\eta, t) = e^{i\Omega t} f(\eta)$ in eqn (10), we get

$$f''(\eta) + P_r(R+1)f'(\eta) - i\Omega \frac{P_r}{4} f(\eta) = 0 \quad (28)$$

Which has to be solved under the boundary conditions,

$$f(0)=1, \quad f(\infty)=0 \quad (29)$$

$$\text{Hence } f(\eta) = e^{-\frac{\eta}{2}[P_r(R+1)+R_1 \cos \frac{\beta_1}{2} + iR_1 \sin \frac{\beta_1}{2}]}$$

$$\theta(\eta, t) = e^{i\Omega t - \frac{\eta}{2}[P_r(R+1)+R_1 \cos \frac{\beta_1}{2} + iR_1 \sin \frac{\beta_1}{2}]} \quad (30)$$

Separating real and imaginary part, the real part is given by

$$\theta_r(\eta, t) = \cos\left(\Omega t - \frac{\eta}{2} R_1 \sin \frac{\beta_1}{2}\right) \cdot e^{-\frac{\eta}{2}[P_r(R+1)+R_1 \cos \frac{\beta_1}{2}]} \quad (31)$$

Putting $C(\eta, t) = e^{i\Omega t} g(\eta)$ in eq (11), we get

$$g''(\eta) + S_c g'(\eta) - g(\eta) \left\{ \frac{4Q + i\Omega}{4} \right\} S_c = 0 \quad (32)$$

Which can be solved under boundary conditions

$$g(0) = 1, g(\infty) = 0$$

$$\text{Hence, } g(\eta) = e^{-\frac{\eta}{2}[S_c + R_2 \cos \frac{\beta_2}{2} + iR_2 \sin \frac{\beta_2}{2}]}$$

$$C(\eta, t) = e^{i\Omega t - \frac{\eta}{2}[S_c + R_2 \cos \frac{\beta_2}{2} + iR_2 \sin \frac{\beta_2}{2}]} \quad (33)$$

Separating real and imaginary part, the real part is given by

$$C_r = \left\{ \cos\left(\Omega t - \frac{\eta}{2} R_2 \sin \frac{\beta_2}{2}\right) e^{-\frac{\eta}{2}[S_c + R_2 \cos \frac{\beta_2}{2}]} \right\} \quad (34)$$

In order to solve equation (26),

Substituting

$$e^{i\Omega t} F(\eta) \text{ and using boundary condition } \psi =$$

$$F(0) = e^{i\Omega t}$$

$$F(\infty) = 0 \quad (35)$$

Separating real and imaginary part, we get

$$u = e^{-a_4 \eta} [\cos \eta a_5 + \{(A_5 A_7 + A_8 A_{10}) \cos(\Omega t - \eta a_5)\} + (A_6 A_7 + A_9 A_{10}) \sin(\Omega t - \eta a_5)] - e^{-a_6 \eta} [A_5 A_7 \cos(\Omega t - \eta a_7) + A_6 A_7 \sin(\Omega t - \eta a_7)] - e^{-a_8 \eta} [A_8 A_{10} \cos(\Omega t - \eta a_9) + A_9 A_{10} \sin(\Omega t - \eta a_9)] \quad (36)$$

$$\omega = e^{-a_4 \eta} [\{\sin \eta a_5 + \{(A_5 A_7 + A_8 A_{10}) \sin(\Omega t - \eta a_5)\} - (A_6 A_7 + A_9 A_{10}) \cos(\Omega t - \eta a_5)\}] - e^{-a_6 \eta} [A_5 A_7 \sin(\Omega t - \eta a_7) - A_6 A_7 \cos(\Omega t - \eta a_7)] - e^{-a_8 \eta} [A_8 A_{10} \sin(\Omega t - \eta a_9) - A_9 A_{10} \cos(\Omega t - \eta a_9)] \quad (37)$$

Results and Discussion

The effect of hall current on MHD free convection flow of elastico-viscous fluid past an impulsively started

infinite vertical plate with mass transfer has been carried out in preceding sections. In order to get physical insight into the problem, the velocity, temperature, concentration fields, shear stress, rate of heat and mass transfer have been discussed by assigning numerical value to M (magnetic parameter), m (Hall parameter), α (Non-Newtonian parameter), Sc (Schmidt number) and Ω (frequency parameter). The values of Prandtl number (Pr), are taken equal to 3 and 10 which represent the saturated liquid (Freon CF_2Cl_2) at 272.3 K and Gasoline at 1 atm. pressure and 20°C respectively. The value of Grashof number (G) and modified Grashof number (Gc) are taken equal to 5 and 2 respectively.

Figures 1.2 -1.5 gives the velocity component u, for $G = 5.0$, $G_c = 2.0$ and $Pr = 3$ taking different values of M, m, R, Q and α . It is observed that an increase in the applied magnetic field parameter (M), Hall parameter ($m = \omega_e \tau_e$), Radiation parameter (R) and chemical reaction parameter (Q) leads to a decrease the velocity component while reverse effect is observed for non-Newtonian fluid (α). Figure 2.1-2.5 represents the velocity component w, for $G = 5$, $G_c = 2$ and $Pr = 3$. An increase in applied magnetic field (M), Hall parameter (m) and chemical reaction parameter (Q) decreases the velocity in z-direction, whereas increase in radiation parameter (R) and non-Newtonian fluid (α) increases the velocity. Here it is observed that the maximum velocity occurs near the plate and as $n \rightarrow \infty$ the velocity profiles terminate to zero. The temperature profiles have been shown in figure 4 for $Pr = 3$. The temperature distribution increases with increase in frequency (Ω) while reverse phenomenon is observed for R and Prandtl number Pr. The maximum value occurs near the plate and then decreases far away from the plate. Figure 4 represents the concentration profiles for Helium ($Sc = 0.30$) and Ammonia ($Sc = 0.78$). The figure reveals that an increase in Sc leads to decrease the concentration distribution because the smaller values of Sc are equivalent to increasing the chemical molecular diffusivity. Further it is observed that the concentration decreases due to increasing values of the chemical parameter. This shows that the diffusion rates can be tremendously altered by chemical reactions.

Knowing the velocity field it is important from a practical point of view to know the effect of physical parameters, Sc, M, m and α on skin friction. We now calculate the skin friction from these relations

$$\tau_{xw} = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$$

In non-dimensional form it takes

$$\tau_1 = \left(\frac{\partial u}{\partial \eta} \right)_{\eta=0}$$

Where τ_1 is the x-component of skin friction.

Similarly z- component of skin friction τ_2 is given as

$$\tau_2 = \left(\frac{\partial w}{\partial \eta} \right)_{\eta=0} \text{ (in non-dimensional form).}$$

The shearing stress along x-axis τ_1 is shown in Figure 5.1- 5.3 for different values of M, R and chemical reaction (Q). It is observed that there is a rise in τ_1 with increasing radiation parameter but fall with increasing values of applied magnetic field parameter M and chemical reaction parameter. Figure 6.1-6.3 represents the shearing stress along z-axis (τ_2). From this we concluded that increase in M and radiation parameter R decreases the skin friction τ_2 while reverse effect is observed for chemical reaction parameter (Q)

The rate of heat transfer in terms of Nusselt number is given by

$$Nu = \frac{qv}{v_0 K} (T_w - T_\infty)$$

where $q = -K \left. \frac{\partial T}{\partial y} \right|_{y=0}$

In non-dimensional form it is given by

$$Nu = - \left(\frac{\partial \theta}{\partial \eta} \right)_{\eta=0} = \frac{1}{2} \left[Pr(R + 1) \cos \Omega t + R_1 \cos \left(\Omega t + \frac{\beta_1}{2} \right) \right] \quad (38)$$

(in non-dimensional form).

The rate of mass transfer is given by

$$J^* \text{ (Diffusion flux)} = -\rho D \left. \frac{\partial C^*}{\partial y} \right|_{y=0}$$

The coefficient of mass transfer which is generally known as Sherwood number S_h is given by

$$S_h = \frac{J^* v}{v_0 \rho D (C_w - C_\infty)} = \left. \frac{\partial C}{\partial \eta} \right|_{y=0}$$

$$S_h = \frac{1}{2} \left[Sc \cos \Omega t + R_2 \cos \left(\Omega t + \frac{\beta_2}{2} \right) \right] \quad (39)$$

Numerical values of heat and mass transfer rate are calculated from equation (38) and (39) and these values are plotted in Figure 8 and 9. Figure 8 gives the rate of heat transfer for $Pr = 3$ and $Pr = 10$ against the radiation

parameter (R) and Prandtl number (Pr). It is noticed that an increase in Prandtl number increases the rate of heat transfer whereas increase in radiation parameter decreases Nu. The rate of mass transfer (S_h) for different

values of Sc is shown in Figure 9. It is concluded that Sherwood number decreases with increasing Schmidt number but rate of mass transfer (S_h) increases with increasing values of chemical reaction parameter (Q).

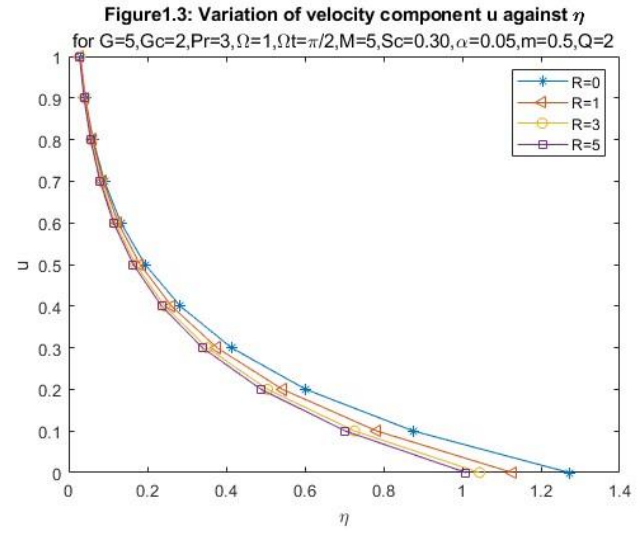
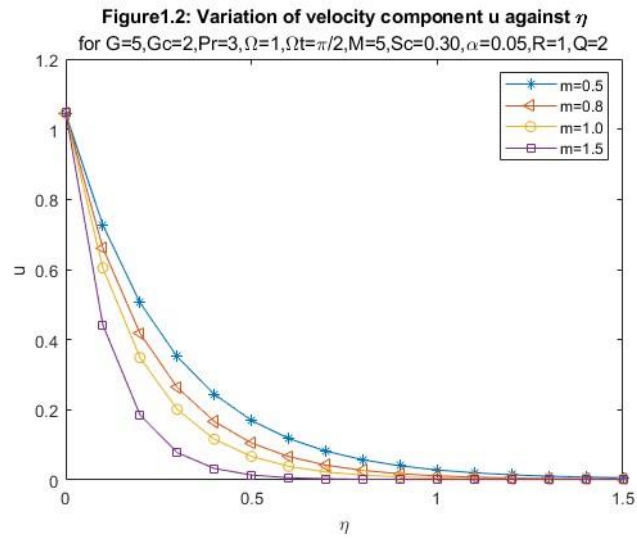
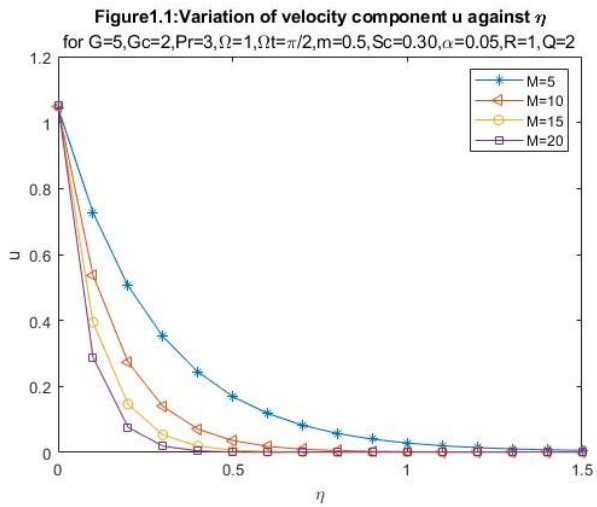


Figure 1.4: Variation of velocity component u against η
 for $G=5, G_c=2, Pr=3, \Omega=1, \Omega t=\pi/2, M=5, Sc=0.30, \alpha=0.05, m=0.5, R=1$

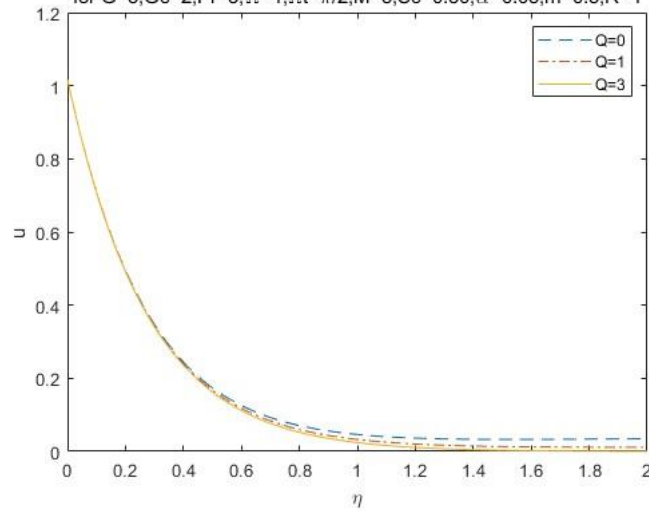


Figure 1.5: Variation of velocity component u against η
 for $G=5, G_c=2, Pr=3, \Omega=1, \Omega t=\pi/2, M=5, Sc=0.30, Q=2, m=0.5, R=1$

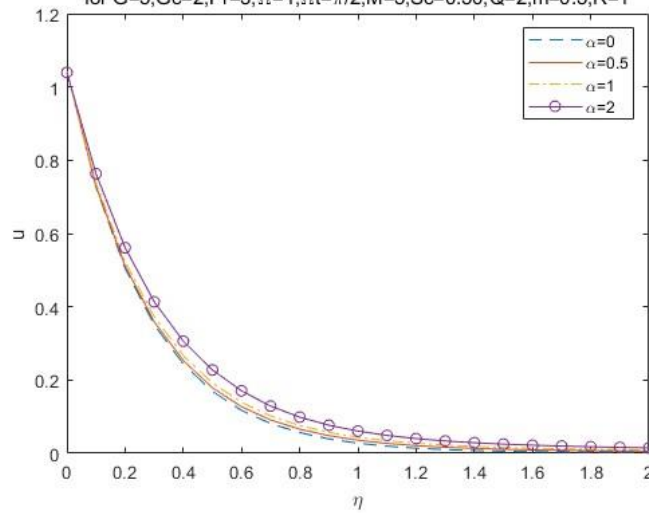


Figure 2.1: Variation of velocity component w against η
 for $G=5, G_c=2, Pr=3, \Omega=1, \Omega t=\pi/2, m=0.5, Sc=0.30, \alpha=0.05, R=1, Q=2$

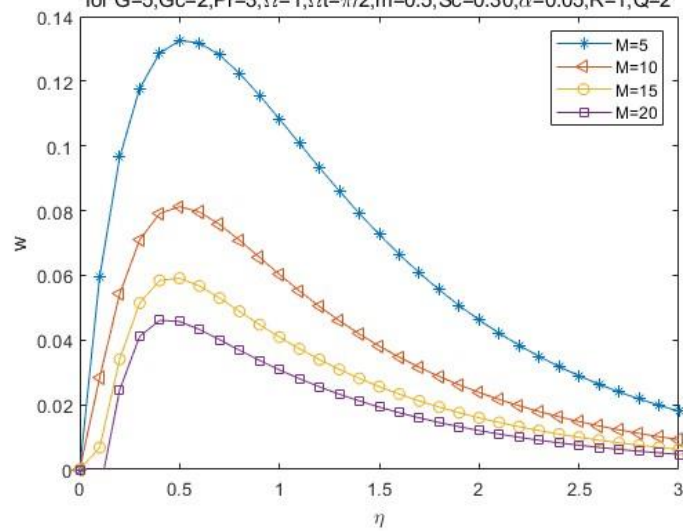


Figure 2.2: Variation of velocity component w against η
 for $G=5, G_c=2, Pr=3, \Omega=1, \Omega t=\pi/2, M=5, Sc=0.30, \alpha=0.05, R=1, Q=2$

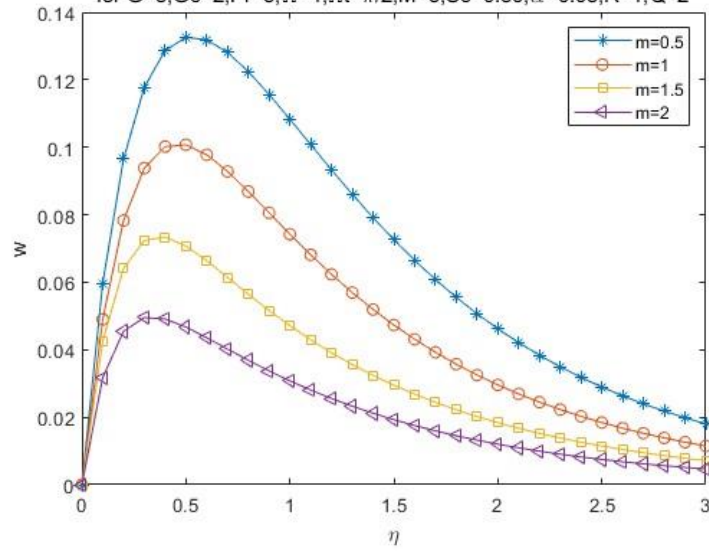


Figure 2.3: Variation of velocity component w against η
 for $G=5, G_c=2, Pr=3, \Omega=1, \Omega t=\pi/2, M=5, Sc=0.30, \alpha=0.05, m=0.5, Q=2$

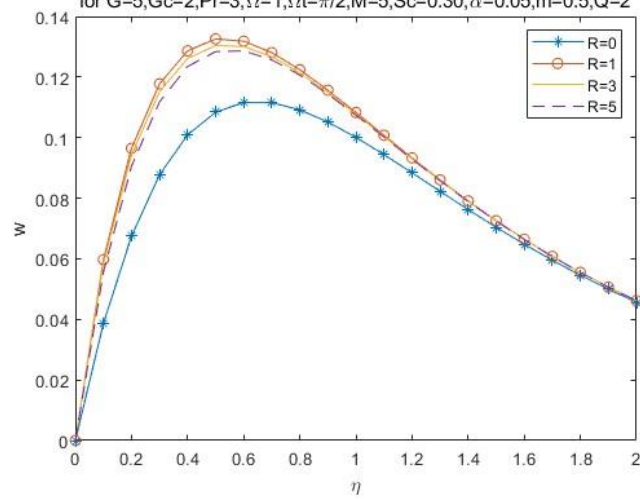


Figure 2.4: Variation of velocity component w against η
 for $G=5, G_c=2, Pr=3, \Omega=1, \Omega t=\pi/2, M=5, Sc=0.30, \alpha=0.05, m=0.5, R=1$

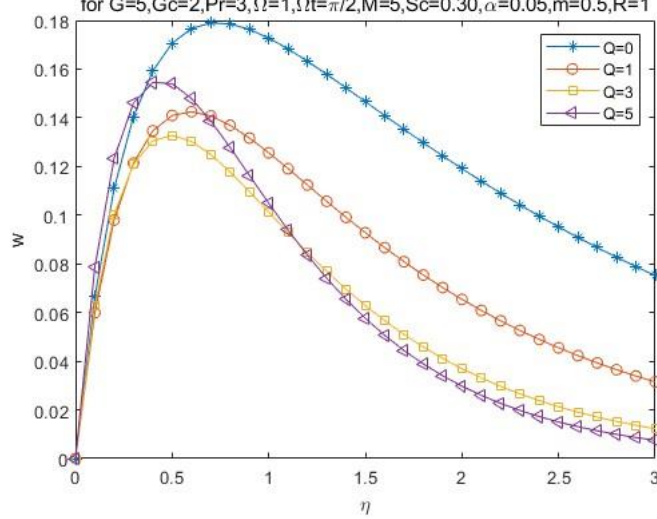


Figure 2.5: Variation of velocity component w against η
 for $G=5, G_c=2, Pr=3, \Omega=1, \Omega t=\pi/2, M=5, Sc=0.30, Q=2, m=0.5, R=1$

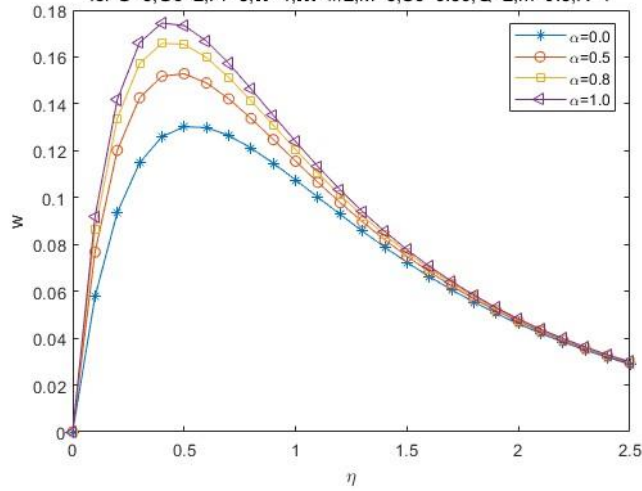


Figure 3: variation of temperature θ_r against η

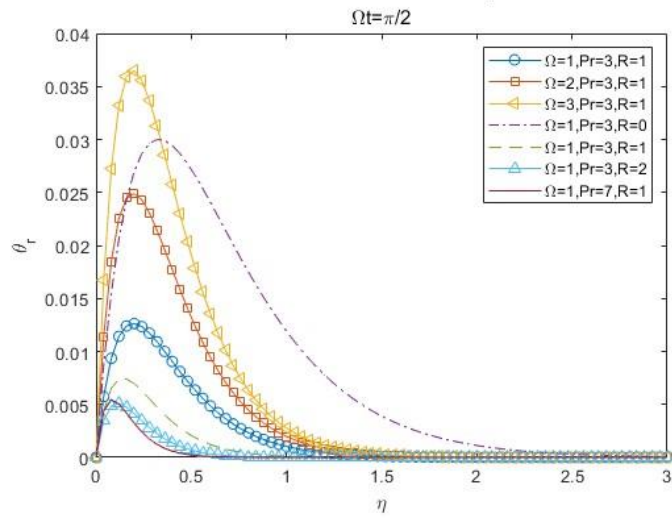


Figure 4: variation of concentration C_r against η

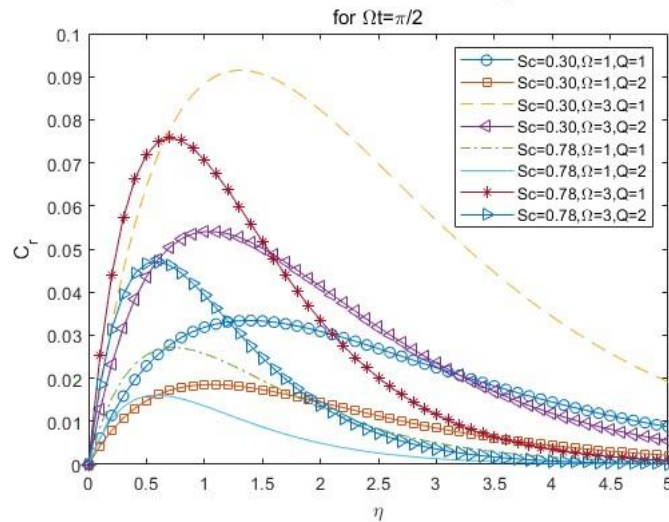


Figure 5.1: Variation of shearing stress τ_1 against m , for $G=5, Gc=2, Pr=3, \Omega=1$

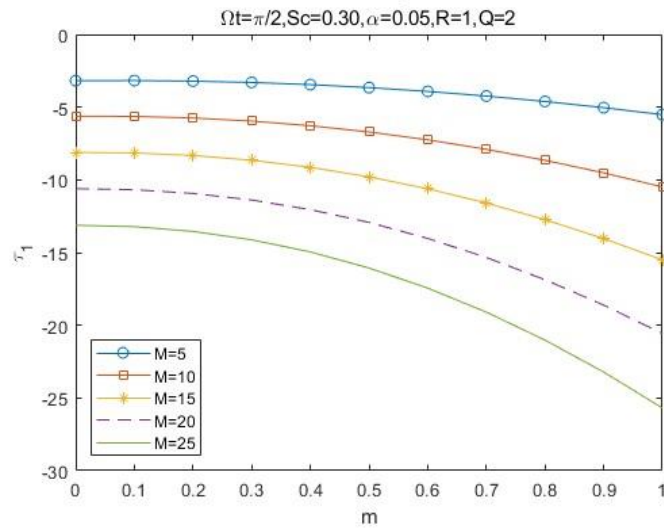


Figure 5.2: Variation of shearing stress τ_1 against m , for $G=5, Gc=2, Pr=3, \Omega=1$

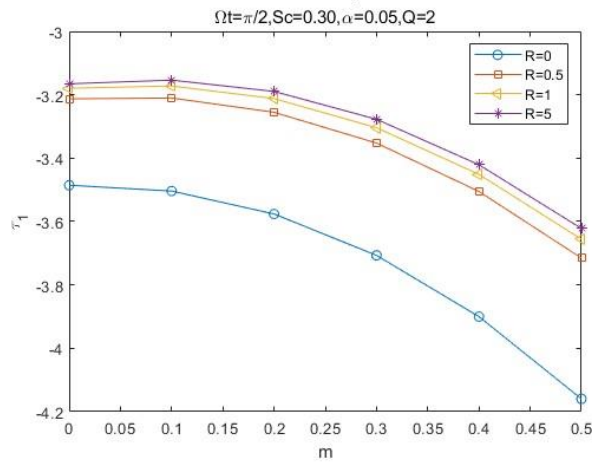


Figure 5.3: Variation of shearing stress τ_1 against m , for $G=5, Gc=2, Pr=3, \Omega=1$

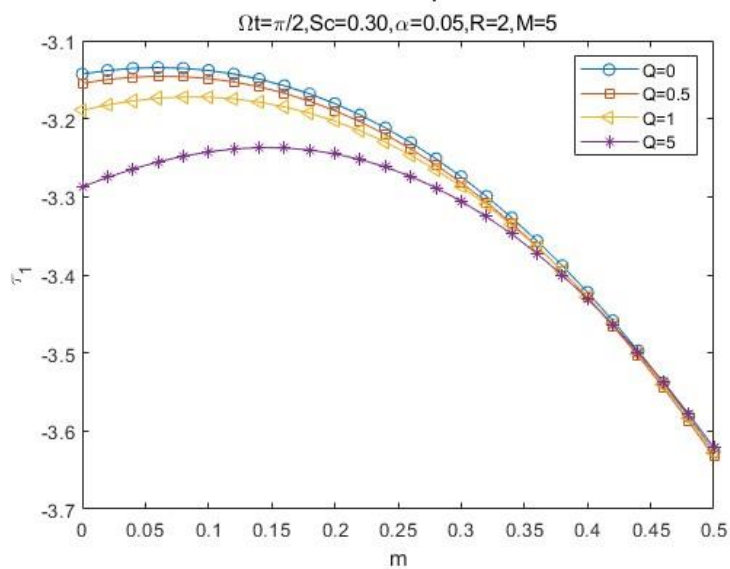


Figure 6.1: Variation of shearing stress τ_2 against m , for $G=5, Gc=2, Pr=3, \Omega=1$

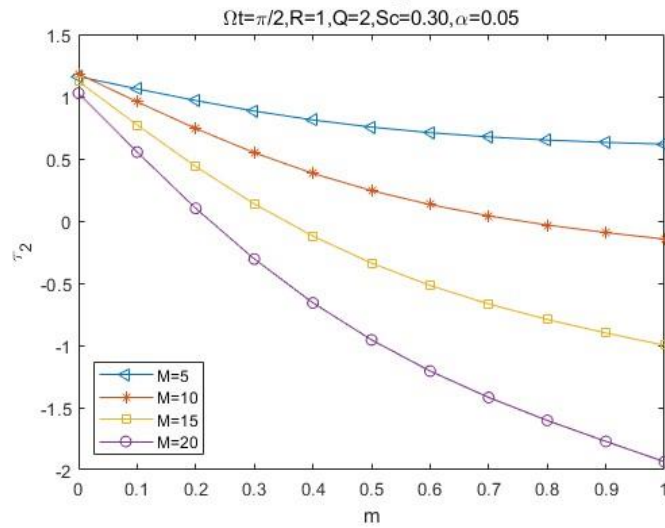


Figure 6.2: Variation of shearing stress τ_2 against m , for $G=5, Gc=2, Pr=3, \Omega=1$

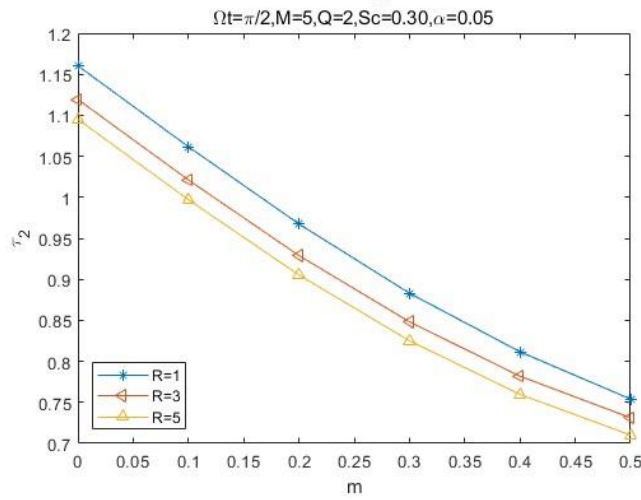


Figure 6.3: Variation of shearing stress τ_2 against m , for $G=5, Gc=2, Pr=3, \Omega=1$

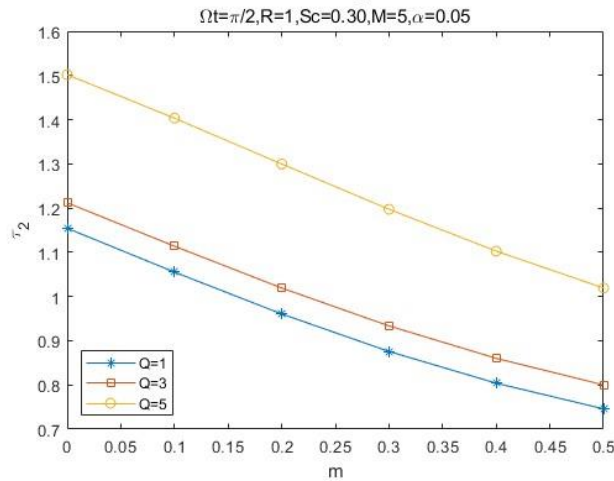


Figure 7: Rate of heat transfer for $\Omega t = \pi/2$ against frequency Ω

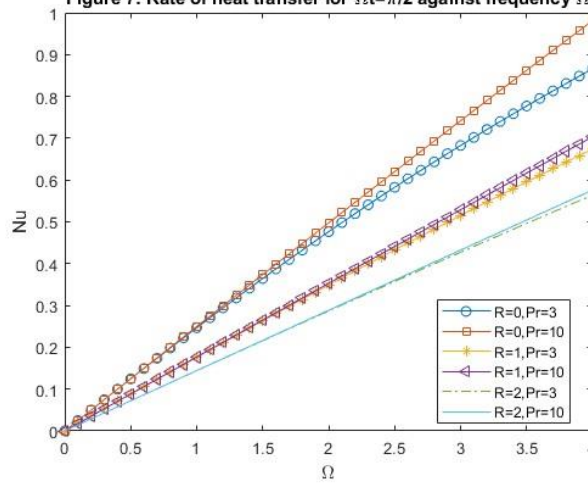
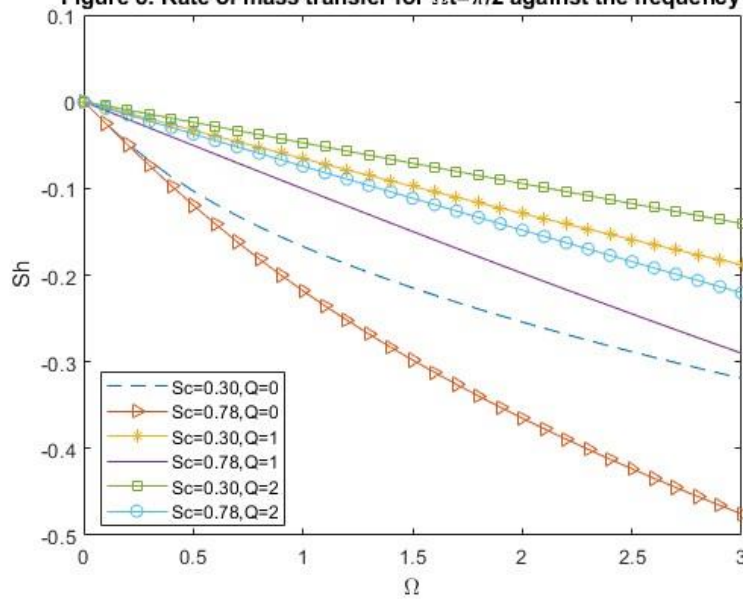


Figure 8: Rate of mass transfer for $\Omega t = \pi/2$ against the frequency Ω



APPENDIX

$$a_1 = \frac{\alpha\Omega}{4},$$

$$A_1 = 4(a_2 + a_1 a_3),$$

$$A_3 = 1 + r^{1/4} \cos \frac{\gamma}{2}$$

$$r = (1 + A_1^2)^2 + A_2^2,$$

$$a_4 = \frac{A_3 - a_1 A_4}{2(1 + a_1^2)},$$

$$a_6 = \frac{1}{2} [Pr(R + 1) + R_1 \cos \frac{\beta_1}{2}]$$

$$a_7 = \frac{1}{2} R_1 \sin \frac{\beta_1}{2}$$

$$a_8 = \frac{1}{2} [Sc + R_2 \cos \frac{\beta_2}{2}]$$

$$a_9 = \frac{1}{2} R_2 \sin \frac{\beta_2}{2}$$

$$a_2 = \frac{M}{4(1 + m^2)},$$

$$A_2 = 4(a_3 - a_1 a_2)$$

$$A_4 = r^{1/4} \sin \frac{\gamma}{2}$$

$$\gamma = \tan^{-1} \frac{A_2}{1 + A_1}$$

$$a_5 = \frac{a_1 A_3 + A_4}{2(1 + a_1^2)},$$

$$a_3 = \left(\frac{\Omega}{4} - \frac{Mm}{4(1 + m^2)} \right)$$

$$R_1 = \{P_r^{1/2}(P_r^2(R+1)^2 + \Omega^2)^{1/4}\}$$

$$R_2 = [S_c^{1/2}\{(S_c^2 + 4Q)^2 + \Omega^2\}^{1/4}]$$

$$\beta_1 = \tan^{-1} \frac{\Omega}{P_r(R+1)}, \quad \beta_2 = \tan^{-1} \frac{\Omega}{S_c + 4Q}$$

$$A_5 = a_6^2 - a_7^2 + 2a_1a_6a_7 - a_6 - a_2$$

$$A_6 = 2a_6a_7 - a_1(a_6^2 - a_7^2) - a_7 - a_3,$$

$$A_7 = \frac{G}{4[A_5^2 + A_6^2]}$$

$$A_8 = a_8^2 - a_9^2 + 2a_1a_8a_9 - a_8 - a_2$$

$$A_9 = 2a_8a_9 - a_1(a_8^2 - a_9^2) - a_9 - a_3, \quad A_{10} = \frac{G_c}{4[A_8^2 + A_9^2]}$$

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