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Application of Edge Detour Monophonic Number of a Fuzzy Graph

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Abstract: In networks with uncertain or unreliable links, fuzzy graphs model link reliability. The edge detour monophonic number helps identify crucial nodes for secure detour routing when shortest paths are not viable. Let M be a set of vertices in a connected non-trivial fuzzy graph $G: (V, \sigma, \mu)$. Then the edge detour monophonic closure of M, denoted by $(dM)_e$, is the set of all edges lying on the fuzzy detour monophonic path between a pair of vertices of M. A set M of vertices in a connected non-trivial fuzzy graph $G: (V, \sigma, \mu)$ is defined to be an edge detour monophonic set of G if $(dM)_e = E(G)$, the edge set of G. An edge detour monophonic set of minimum cardinality is called an edge detour monophonic basis of G and the cardinality of an edge detour monophonic basis in G is the edge detour monophonic number of G, denoted by $dmn_e(G)$. The minimum number of vertices in a set such that every edge in the fuzzy graph lies in a monophonic detour path between two vertices in the set. In this study, the applications of edge detour monophonic number of fuzzy graph in the system of transporting goods are applied in order to optimize the number of commercial vehicle inspectors required to patrol and inspect the vehicle routes currently used in an urban road network.

Keywords: *fuzzy graph, fuzzy strong chord, fuzzy monophonic path, fuzzy detour monophonic path, edge detour monophonic number.*

AMS Subject Classification: 05C72, 05C38, 05C90.

1. Introduction

In the year 1736, Euler first proposed the idea of graph theory. The concept of fuzzy sets was first described by L.A. Zadeh [17] in 1965, and it has since been successfully used to resolve a variety of real-world decision issues that are frequently ambiguous. A fuzzy set is a generalization of a crisp set in which the set's components may or may not be members. Instead of merely considering 0 or 1, fuzzy sets allow their members to have membership degree values between 0 and 1 for better results. The membership degree of a member, which is a single value within the range [0, 1], does not equal probability; rather, it represents the element's degree of belongingness to the fuzzy set. Due to ignorance of the issue, the single membership degree values, however, are unable to handle the uncertainties. Rosenfeld [9] first proposed the idea of fuzzy graph in 1975. The fuzzy graph theoretic terminologies refer Pal et al. [8]. Narayan and Sunitha [7] introduced the concept of connectivity in fuzzy graph and its complement. If $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for every $x, y \in \sigma^*$, a fuzzy graph $G: (V, \sigma, \mu)$ is a fuzzy graph. Strong arc concepts and fuzzy end node concepts are introduced by Bhutani and Rosenfeld [3]. These arcs are further classified as α, β, δ and δ^* arcs by Sunil Mathew and Sunitha [5]. The fuzzy cycles and trees are introduced by Mordeson and Yao [6]. The term generalized fuzzy graph was introduced by Samanta and Sarkar [10]. A path P is a sequence of different vertices of length n such that $\mu(u_{i-1}, u_i) > 0, i = 1,2, \ldots, n$ and the degree of membership of a weakest arc is defined as its strength. If $u_0 = u_n, n \geq 3$, the path turns into a cycle, and if the cycle contains more than one weakest arc, it is referred to be a fuzzy cycle. The maximum strength of all paths connecting two vertices x and y is known as the strength of connectedness, and it is shown by the

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symbol $CONN_G(x,y)$. When an arc in a fuzzy graph is eliminated, its weight must at least equal the connectedness of its end vertices. An $x-y$ path is said to be a strong path if Ponly comprises strong arcs. If a connected fuzzy graph $G:(V,\sigma,\mu)$ has a spanning fuzzy sub graph $F:(V,\sigma,\upsilon)$ a tree where for every arcs (x,y) not in F, $CONN_F(x,y) > \mu(x,y)$ it is referred to as a fuzzy tree. A fuzzy tree with a singular maximum spanning tree that is a star is called a fuzzy star. If a fuzzy sub graph $H:(V,\tau,\pi)$ is bipartite and spans the fuzzy graph $G:(V,\sigma,\mu)$ then the weight of the pair (x,y) in G is strictly less than the strength of the pair (x,y) in H for all edges that are not in H. In 2011, Al-Haway [1] defined the complete fuzzy graph. If every vertex of a fuzzy bipartite graph with fuzzy bipartite (V_1,V_2) is a strong neighbor to every node of V_1, the graph is said to be a complete fuzzy bipartite.

In crisp graph theory, Santhakumaran and John [13] studied the concept of edge geodetic number of a graph. In 2019, Sameeha Rehmani & Sunitha [12] introduced the concept of edge geodesic number of a fuzzy graph. In 2025, Sameeha Rehmani [11] developed the term Perfect S-geodetic fuzzy graph. Balaraman et al. [2] introduced the concepts of geodetic dominating sets in FG and also introduced new vulnerability parameter geodetic domination integrity. It is also applied in the telecommunication network system. A crisp graph's detour monophonic number was first introduced by Titus et al. [16]. John and Arul Paul Sudhahar [4] introduced the concept of edge monophonic number of a graph. In a crisp graph, the chord of a path P is the edge that connects two of its non-adjacent vertices. If a path P lacks chords, it is said to be monophonic path. The term " $x-y$ detour monophonic path" refers to the longest $x-y$ monophonic path. If each vertex v of a graph G lies on an $x-y$ detour monophonic path for some $x,y \in M$, then the set $M$of those vertices is a detour monophonic set. The detour monophonic number of G is the smallest cardinality of a detour monophonic set of G. A set M of vertices is said to be an edge monophonic set of G if every edge of G lies on a monophonic path connecting some pair of vertices in M. The minimum cardinality of G's edge monophonic sets is known as the edge monophonic number. The edge detour monophonic number of crisp graph was introduced by Santhakumaran et al. [14, 15]. A set $M$of vertices that has every edge of G lying on a detour monophonic path connecting some pair of vertices in M is known as an edge detour monophonic set of G.The lowest cardinality of G's edge detour monophonic sets is known as its edge detour monophonic number, which is represented by the symbol $edm(G)$.

We introduced and looked into detour monophonic sets in a fuzzy graph as a result of the edge detour monophonic number of a crisp graph. Let $P: u_1, u_2, \dots, u_i, u_{i+1}, \dots, u_j, u_{j+1}, \dots, u_n$ be a path of a fuzzy graph $G:(V,\sigma,\mu)$. A strong chord of a path in a fuzzy graph is an edge $u_i u_j$ if $\mu(u_i,u_j) \geq \mu(u_i,u_{i+1}) \wedge \mu(u_{i+1},u_{i+2}) \wedge \dots . \wedge \mu(u_{j-1},u_j)$. A path P in a fuzzy graph is called fuzzy monophonic path if it is a strong chord less path.

In this study, we introduce the idea of a fuzzy graph's edge detour monophonic number and its applications of goods transportation system. Even though all the vertices are covered by the network when studying detour monophonic sets, some of the edges may be overlooked while creating the channel for a communication network. In the case of edge detour monophonic sets, this flaw is solved, making edge detour monophonic sets more beneficial for actual applications of communication networks. In order to minimize the loss experienced by various transportation corporations due to a lack of collection demonstrated, edge detour monophonic sets are applied in order to optimize the number of commercial vehicle inspectors required to patrol and inspect the vehicle routes currently used in an urban road network and, concurrently, to identify those routes receiving less priority and, therefore, eliminate them.

2. Application of the Edge Detour Monophonic Number in Goods Transportation System

In India, one of the most popular modes of transportation is road travel. Road transport is used every day around the nation to move countless quantities of commodities from one location to another. Trucks, trailers, and carriers transport things around the clock to their final destinations. Road transport has various advantages, including low investment requirements, door-to-door delivery, flexibility in service, suitability for short distances, and advantages for businesses. The main advantages of road transport include the ability to convey goods and materials from door to door and the ability to load and unload cargo at very reasonable prices. Compared to other forms of transport like trains and the air, road transport requires a fairly small amount of expenditure. Additionally, compared to railroads, the cost of building, operating, and maintaining roads is lower. Road transport is a reliable means of moving daily necessities including food, medicine, and agricultural products from one location to another. You cannot even begin to think how you would get by without access to the road for a normal existence.

One of the basic problems in graph theory is the covering problem, which has been applied to fuzzy graphs in the form of fuzzy vertex covering problems, fuzzy edge covering problems, fuzzy minimal weight edge covering problems, and others. Path coverings, particularly those involving shortest paths and geodesics, make up a significant subclass of fuzzy covering problems. In this paper, we introduce and investigate a related problem that seeks to minimize the loss resulting from non-collection by various transport corporations by maximizing the number of commercial vehicle inspectors needed to patrol and inspect the commercial vehicle route prevalent in the urban road network. At the same time, we identify and eliminate those routes receiving lower priority among goods.

A fuzzy graph is used to describe the urban road network, with the vertices being the goods stations and the edges being the possible commercial vehicle routes connecting these stations. Vertices and edges have the following established membership values.

2.1 Membership Values of Vertices

Before discussing the vertices' membership values, the associated word "station-capacity" $\check{z}$ is defined. It specifies the most commercial vehicles that can be placed on the track in a goods station at a given moment. N stands for it. N might not be the same for every goods station. However, it ought to be predetermined for a specific goods station. Thus, the ratio of the number of commercial vehicles entering the goods station to N is used to establish the membership value of a goods station.

Let $V = \{S_1, S_2, \dots, S_m\}$ represent the available goods station along a certain metropolitan road system. Let $N_1, N_2, \dots, N_m$ represent the individual station capabilities for each goods station.

Define the mapping $\sigma : V \rightarrow [0, 1]$ such that

$$\sigma(S_k) = \begin{cases} \frac{E_k}{N_K} \, ; if \ E_k \leq N_k \\ 1 \, ; if E_k \ > N_k \end{cases} \qquad (1 \ \leq k \ \leq m)$$

(1)

Where, E_k reflects the quantity of commercial vehicles entering the station S_k at a specific time period.

2.2 Membership Values of Edges

An urban road network's edges each correspond to a section of a specific commercial vehicle route. A similar word "satisfied goods number" $\check{z}$ is defined before membership values of edges are specified.

P , a genuine positive number, is taken as the set number of commodities. The commercial vehicle route is deemed useful if the total number of goods along a specific route exceeds P. The term "satisfied goods number" refers to this set quantity of items.

Let $\mu : V \times V \rightarrow [0,1]$ be a mapping such that

$$(S_i, S_j) = \begin{cases} \frac{p}{P}\left(\sigma(S_i) \wedge \sigma(S_j)\right); if \ p \epsilon [0, P] \\ \sigma(S_i) \wedge \sigma(S_j); \ \ if \ p > P \end{cases}$$

(2)

Where, p is the number of goods along a commercial vehicle route in a specific amount of time and P is the satisfied goods number.

2.3 The Patrolling Problem

The commercial vehicle routes are inspected by commercial vehicle inspectors, and the urban road network is patrolled. We expect the network to be a simple fuzzy graph without lose of generality.

The following requirements are met by a road patrolling plan:

- In the road network, the commercial vehicle route is a detour monophonic.
- One inspector is posted at each end, and two commercial vehicle inspectors patrol the area.
- In the fuzzy graph used to depict the urban road network, $\delta -$ arcs are thought to be vehicle routes with the lowest priority and are therefore not taken into account for patrolling and inspection.
- All bus routes, save those with the lowest priority ($\delta -$arcs), must be examined.

The challenge is to satisfy the above-described plan while minimizing the number of commercial vehicle inspectors needed to patrol and check the vehicle routes in the urban road network.

2.4 An Example of an Urban Road Network as a Fuzzy Graph Model

The collection of commercial vehicle depots in the urban road network shown in Fig. 1.1 is denoted by $V = \{S, S_2, \ldots, S_6\}$.

As a result, the urban road network model is depicted as follows:

Set the "station-capacity" $\check{z}$ of each commercial vehicle station to $N_1, N_2, \ldots, N_6$ (respectively), with the "satisfied goods number" $\check{z}$ set to 30 people each time interval. These values are shown in column 2 of Table 1.

Commercial vehicle inspectors I_1, I_2, I_3, ... etc. patrol the metropolitan road network and check the commercial vehicle routes.

We made the assumption that the commercial vehicle depots are linked to one another by commercial vehicle routes, in accordance with the information in Tables 1 and 2.

Column 4 of Table 1 displays the evaluated σ -values for each station.

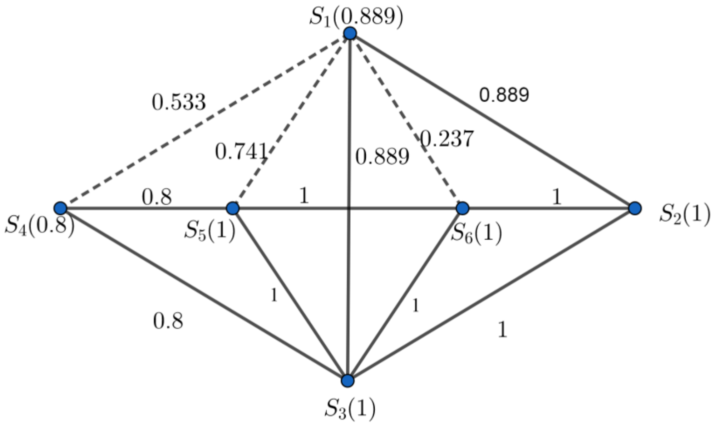

Figure 1. Urban road network (Goods stations)

Table 1. Membership value of goods station in the urban road network

Goods stations	**Station-capacity (N_i)**	**E_i-values**	**σ-values**
S_1	45	40	0.889
S_2	25	30	1
S_3	30	30	1
S_4	50	40	0.8
S_5	35	40	1
S_6	20	25	1

Table 2. Membership values of edges in the urban road network

Edges	**p-values**	**μ-values**
(S_1, S_2)	35	0.889
(S_2, S_3)	33	1

(S_3, S_4)	32	0.8
(S_4, S_1)	20	0.533
(S_1, S_6)	8	0.237
(S_6, S_3)	31	1
(S_3, S_5)	32	1
(S_5, S_1)	25	0.741
(S_1, S_3)	31	0.889
(S_4, S_5)	35	0.8
(S_5, S_6)	36	1
(S_6, S_2)	40	1

Commercial vehicle influx into station S_1 is 40 each time period, while station capacity is 45.

So, $\sigma(S_1) = \frac{40}{45} = 0.889$.

The membership values of additional depots are computed in a similar manner.

After evaluation, the membership values of edges are listed in Table 2.

Table 2 shows that the satisfying goods number for the network is 30 and that the number of goods along the commercial vehicle route $S_1 - S_2$ per period of time is 35. So, $\mu(S_1, S_2) = 0.889 \wedge 1 = 0.889$. Additionally, there are 20 items along the commercial vehicle route $S_4 - S_1$ per time interval, and there are 30 fulfilling things overall. So, $\mu(S_4, S_1) = \frac{20}{30} \times (0.8 \wedge 0.889) = 0.533$.

In the similar manner, membership values for all other network edges are computed and are shown in column 3 of Table 2.

2.5 Optimization using Edge Detour Monophonic Number

The degree to which a commercial vehicle route connecting two goods stations is given priority depends on the distance between the stations. The commercial vehicle routes in Fig. 3.1 that directly link the bus depots (S_1, S_4) , (S_1, S_5) and (S_1, S_6) are given the lowest priority, possibly due to poor road conditions or hazardous physical environmental conditions like a forest area, a Maoist zone, and so on. Out of these three arcs, the arc (S_1, S_6) has very low priority. So this edge is not considered for patrolling and inspection. Thus, the following is a patrolling solution for the network in Figure 1.

- I_1, I_6 patrol commercial vehicle route $S_1 - S_2 - S_6$, $S_1 - S_3 - S_6$ and $S_1 - S_5 - S_6$
- I_2, I_4 patrol commercial vehicle route $S_2 - S_6 - S_5 - 4$ and $S_2 - S_1 - S_5 - S_4$.
- I_1, I_4 patrol commercial vehicle routes $S_1 - S_2 - S_6$ and $S_1 - S_3 - S_6$.
- I_2, I_5 patrol commercial vehicle route $S_2 - S_6 - S_5$, $S_2 - S_3 - S_5$ and $S_2 - S_1 - S_5$
- I_3, I_4 patrol commercial vehicle route $S_3 - S_4$.
- I_4, I_6 patrol commercial vehicle route $S_4 - S_1 - S_2 - S_6$.

Note that the goods stations S_1, S_2, S_3, S_4, S_5 and S_6 are where the traffic inspectors I_1, I_2, I_3, I_4, I_5 and I_6 are situated, respectively. $\{S_1, S_2, S_3, S_4, S_5, S_6\}$ is the edge detour monophonic bases of the fuzzy graph shown in Figure 1. In this way, the edge detour monophonic number aids in determining the bare minimum of commercial vehicle inspectors needed to monitor the urban road network, and the network's fuzziness aids in establishing the importance of each commercial vehicle route to the products. Eliminating routes for commercial vehicles that are given lower priority aids transportation companies in reducing the loss incurred from a lack of collection and assists in promoting alternative transportation options.

2.6 The Patrolling Problem Modified

The road patrolling plan provided in section 2.3

is changed by include the following new clause:

- No more than one commercial vehicle route is given to a single pair of inspectors for commercial vehicles. One commercial vehicle inspector, however, is tasked with patrolling additional commercial vehicle routes with other inspectors.

The following criteria are used to assess if the patrolling solution for the optimization problem of the urban road network in Fig.3.1 satisfies the updated patrolling scheme:

- I_1, I_6 patrol commercial vehicle route $S_1 - S_3 - S_6$.
- I_2, I_4 patrol commercial vehicle route $S_2 - S_1 - S_5 - S_4$.
- I_2, I_5 patrol commercial vehicle routes $S_2 - S_3 - S_5$.
- I_3, I_4 patrol commercial vehicle route $S_3 - S_4$.
- I_4, I_6 patrol commercial vehicle route $S_4 - S_1 - S_2 - S_6$.
- I_5, I_6 patrol commercial vehicle routes $S_5 - S_6$.

One pair of inspectors can only be given one commercial vehicle route under this plan. Three pairs of inspectors (I_1, I_6) , (I_2, I_4) and (I_2, I_5)each having two commercial vehicle routes with an equal detour monophonic distance are shown in Figure 1. However, in the patrolling plan, each of these three pairs of inspectors is given a single bus route.

In this instance four commercial vehicle inspectors I_1, I_2, I_3, I_4, I_5 and I_6 are also necessary. They will be stationed in the corresponding goods stations S_1, S_2, S_3, S_4, S_5 and S_6, where $\{S_1, S_2, S_3, S_4, S_5, S_6\}$ is an edge detour monophonic covers of the fuzzy graph shown in Figure 1. It is also an edge detour monophonic basis, though. Therefore, six commercial vehicle inspectors are a minimal requirement to monitor the urban road network.

3. Conclusion

Graph theory is a needful tool for solving real life problems in different areas. Fuzzy graph theory is a new dimension of graph theory which is a useful tool for real life uncertainties problems. In this article we studied the edge detour monophonic number of a fuzzy graph and we have given the applications of edge detour monophonic number of fuzzy graph in the system of transporting goods are applied in order to optimize the number of commercial vehicle inspectors required to patrol and inspect the vehicle routes currently used in an urban road network. In future, we can apply this concept using other decision making problems.

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