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k-vertex Anti-duplication Self Switching of Graphs

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Abstract: For a finite undirected graph $G(V, E)$ and a non-empty set $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and its complement $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. If $G \cong G^{\sigma}$, then σ is called as self switching of G and if $|\sigma| = k$, then it is called as k-vertex self switching. The set of all k-vertex self switchings of G is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$. k-vertex anti-duplication of the k vertices $v_i \in \sigma$ $(1 \leq i \leq k)$ produces a new graph G' by adding new vertices v_i' $(1 \leq i \leq k)$ such that $N_{G'}(v_i') = [N_G[v_i]]^c$ for $1 \leq i \leq k$. This paper explores the characteristics of k-vertex anti-duplication, propose the concept of k-vertex anti-duplication self switching and analyzes its associated properties.

Keywords : k-vertex self switching, anti-duplication, k-vertex anti-duplication switching, k-vertex anti-duplication self switching, $AD(\sigma G)$

AMS Subject Classification : *05C38, 05C60.*

Introduction

Lint and Seidel introduced the concept of switching in 1966. For a finite undirected graph $G(V, E)$ and a non-empty set $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}(V, E')$ which is obtained from G by removing all edges between σ and its complement $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. Switching, also known as Seidel switching or $|\sigma|$-vertex switching, has been explained by Seidel [1, 6]. If $\sigma = \{v\} \subset V$, then the corresponding switching $G^{\{v\}}$ is called as vertex switching and is denoted by G^{v}. In 2007, Jayasekaran introduced the idea of self switching [7]. When $G \cong G^{\sigma}$, σ is called as self switching of G. It is sometimes referred to as $|\sigma|$-vertex self switching and if $|\sigma| = k$, then it is called as k-vertex self switching [5]. The set of all k-vertex self switchings of G is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$. Duplication of a vertex v of a graph G produces a new graph G' by adding a new vertex v' such that $N_{G'}(v') = N_G(v)$. Jayasekaran and Prabavathy introduced the idea of duplication self vertex switching [4]. A vertex v is termed duplication self vertex switching of G when v is a self vertex switching of the resultant graph obtained after duplication of v. Jayasekaran and Ashwin Shijo introduced the concept of anti-duplication of a point and anti-duplication self vertex switching [2, 3]. Anti-duplication of a vertex v in G produces a new graph G' by adding a new vertex v' such that

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$N_{G'}(v') = [N_G[v]]^c$. A vertex v is called anti-duplication self vertex switching of a graph G if the resultant graph obtained after anti-duplication of v has v as a self vertex switching.

1 Preliminaries

Definition 2.1. *[4]* ***Duplication of a vertex*** v *of a graph* G *produces a new graph* G' *by adding a new vertex* v' *such that* $N_{G'}(v') = N_G(v)$. *In other words, a vertex* v' *is said to be duplication of* v *if all the vertices which are adjacent to* v *in* G *are also adjacent to* v' *in* G'.

The graph obtained from G *after duplication of a vertex* v *is denoted as* $D(vG)$.

Definition 2.2. *[2]* ***Anti-duplication of a vertex*** v *in* G *produces a new graph* G' *by adding a new vertex* v' *such that* $N_{G'}(v') = [N_G[v]]^c$.

The graph obtained from G *after anti-duplication of the vertex* v *is denoted by* $AD(vG)$.

Definition 2.3. *[3] Let* G *be a graph and let* v' *be the anti-duplication vertex of* v *in* $AD(vG)$. *A vertex* v *is called* ***anti-duplication self vertex switching*** *of a graph* G *if* v *is a self vertex switching of* $AD(vG)$.

Notation 2.4. *In this paper, we use the following notation for our convenience.*

$N_G[v_i] = N_G(v_i) \cup \{v_i\}$ *where* $N_G(v_i)$ *is the neighborhoods of* v_i *in* G.

Theorem 2.5. *[7] Let* $G(V, E)$ *be a graph and let* $\sigma \subset V$ *be a self switching of* G. *Then the number of edges between the vertices of* σ *and* $V - \sigma$ *in* G *is* $\frac{k(p-k)}{2}$ *where* $k = |\sigma|$.

Theorem 2.6. *[5] Let* G *be a graph and let* $\sigma = \{v_1, v_2, \ldots, v_k\} \subset V$ *be a* k*-vertex self switching of* G. *Then for* $k \geq 2$, $\sum_{i=1}^{k} deg_G(v_i) = \frac{k(p-k)}{2} + 2$*(The number of edges between the vertices of* σ *in* G*).*

Corollary 2.7. *[5] For even order graph, there is no odd* k*-vertex self switching.*

2 Main Results

Definition 3.1. *Let* $G(V, E)$ *be a graph and let* $\sigma = \{v_1, v_2, \ldots, v_k\} \subseteq V(G)$. *The* ***k-vertex anti-duplication*** *of the* k *vertices* v_i $(1 \leq i \leq k)$ *produces a new graph* G' *by adding new vertices* v_i' $(1 \leq i \leq k)$ *such that* $N_{G'}(v_i') = [N_G[v_i]]^c$ *for* $1 \leq i \leq k$.

The graph obtained from G *after anti-duplication of the* k *vertices* v_1, v_2,..., v_k *is denoted by* $AD(\{v_1, v_2, \ldots, v_k\}G)$ *or* $AD(\sigma G)$.

Example 3.2. *Consider the graph* G *given in figure 2.1. Let* $\sigma = \{v_1, v_2, v_4\}$.

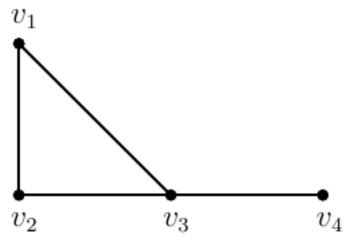

Fig 2.1 G

The 3*-vertex anti-duplication of* $\sigma = \{v_1, v_2, v_4\}$ *is given in figure 2.2.*

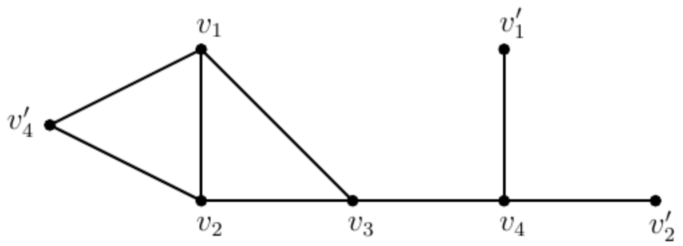

Fig 2.2 $AD(\{v_1, v_2, v_4\}G)$

Theorem 3.3. *Let* G *be a graph and let* $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$. *Then in* $G' = AD(\sigma G)$, $N_{G'}(v_i) \cap N_{G'}(v_i') = \phi$ *for* $1 \leq i \leq k$ *where* v_i' *is the anti-duplication vertex of* v_i.

Proof. Let G be a graph and let $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$. Let v_1', v_2', ..., v_k' be the anti-duplication vertices of $v_1, v_2, \dots, v_k$, respectively. Let $G' = AD(\sigma G)$ and let v_i be any vertex in σ. By definition 3.1, $N_{G'}(v_i') = [N_G[v_i]]^c$. Since $N_G(v_i) \cap [N_G[v_i]]^c = \phi$, $N_G(v_i) \cap N_{G'}(v_i') = \phi$. In G', the vertex v_i is adjacent to the vertices in $N_G(v_i)$ and the anti-duplication vertices of $\alpha = \{v_j \in \sigma : j \neq i$ and v_j is non-adjacent to v_i in $G\}$. This implies that $N_{G'}(v_i) = N_G(v_i) \cup \{v_j' : v_j \in \sigma, j \neq i$ and v_j is non-adjacent to v_i in $G\}$. Now, $N_{G'}(v_i) \cap N_{G'}(v_i') = [N_G(v_i) \cup \{v_j' : v_j \in \sigma, j \neq i$ and v_j is non-adjacent to v_i in $G\}] \cap N_{G'}(v_i') = [N_G(v_i) \cap N_{G'}(v_i')] \cup [\{v_j' : v_j \in \sigma, j \neq i$ and v_j is non-adjacent to v_i in $G\} \cap N_{G'}(v_i')] = \phi \cup [N_{G'}(v_i') \cap \{v_j' : v_j \in \sigma, j \neq i$ and v_j is non-adjacent to v_i in $G\}] = N_{G'}(v_i') \cap \{v_j' : v_j \in \sigma, j \neq i$ and v_j is non-adjacent to v_i in $G\}$. Since all the anti-duplication vertices v_i' $(1 \leq i \leq k)$ are not adjacent to each other in G', $N_{G'}(v_i')$ does not contain $v_j', j \neq i$. This implies that $N_{G'}(v_i') \cap \{v_j' : v_j \in \sigma, j \neq i$ and v_j is non-adjacent to v_i in $G\} = \phi$. Hence, $N_{G'}(v_i) \cap N_{G'}(v_i') = \phi$ for $1 \leq i \leq k$.

Theorem 3.4. *Let* G *be a graph and let* $\sigma \subseteq V(G)$. *If no two vertices of* σ *are adjacent to each other in* G, *then* $AD(\sigma G) - \sigma \cong G^\sigma$ *where* G^σ *is the graph obtained by switching the vertices of* σ *in* G.

Proof. Let G be a graph and let $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$. Let v_1', v_2', ..., v_k' be the anti-duplication vertices of v_1, v_2,..., v_k, respectively. Let v_i be any vertex in σ and let v_j be any vertex in $V - \sigma$. Now, $V(AD(\sigma G) - \sigma) = \{v_j : v_j \in V - \sigma\} \cup \{v_i' : v_i \in \sigma\}$ and $V(G^\sigma) = \{v_i : v_i \in \sigma\} \cup \{v_j : v_j \in V - \sigma\}$. The graph $G' = AD(\sigma G)$ is obtained by adding new vertices v_i' $(1 \leq$

$i \leq k)$ such that $N_{G'}(v_i') = [N_G[v_i]]^c$. That is, in $AD(\sigma G)$, the anti-duplication vertices v_i' are adjacent only to the vertices non-adjacent to v_i in G. Since no two vertices of σ are adjacent to each other in G, they are also not adjacent to each other in G^σ and by definition, the anti-duplication vertices are not adjacent to each other in $AD(\sigma G)$. Now, we define a map $f: V(G^\sigma) \to V(AD(\sigma G) - \sigma)$ by $f(v_i) = v_i'$, $f(v_j) = (v_j)$. Clearly, f is an isomorphism between G^σ and $AD(\sigma G) - \sigma$ and hence $AD(\sigma G) - \sigma \cong G^\sigma$.

Theorem 3.5. *Let G be a (p, q) graph and let $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$. Let v_1', v_2', ..., v_k' be the anti-duplication vertices of v_1, v_2, ..., v_k, respectively. Let $G' = AD(\sigma G)$ be a (p', q') graph. Then*

(i) $deg_{G'}(v_i) = k - 1 +$ number of adjacent vertices of v_i in $V - \sigma$ of G

(ii) For $v \in V - \sigma$, $deg_{G'}(v) = k +$number of vertices in $V - \sigma$ which are adjacent to v in G

(iii) $deg_{G'}(v_i') = p - 1 - deg_G(v_i)$

(iv) $p' = p + k$

(v) $q' = q + k(p-1) - \sum_{i=1}^{k} deg_G(v_i)$.

Proof. Let G' be a graph $AD(\sigma G)$ with p' vertices and q' edges.

(i) By definition, $deg_{G'}(v_i) = deg_G(v_i)$ +number of vertices non-adjacent to v_i in $G[\sigma] = deg_G(v_i) + k - 1$ −number of vertices adjacent to v_i in $G[\sigma] = deg_G(v_i) + k - 1 - |N_{G[\sigma]}(v_i)| = deg_G(v_i) + k - 1 - (deg_G(v_i) -$ number of vertices in $V - \sigma$ of G which are adjacent to v_i in G). Hence, $deg_{G'}(v_i) = k - 1$ +number of vertices adjacent to v_i in $V - \sigma$ of G.

(ii) Let v be a vertex in $V - \sigma$. By definition, $deg_{G'}(v) = deg_G(v) +$ the number of vertices in σ which are non-adjacent to v in $G = deg_G(v) + k -$ number of vertices in σ which are adjacent to v in $G = deg_G(v) + k - (deg_G(v) -$ number of vertices in $V - \sigma$ which are adjacent to v in $G) =$ $k +$ number of vertices in $V - \sigma$ which are adjacent to v in G.

(iii) Since $N_{G'}(v_i') = [N_G[v_i]]^c$, $deg_{G'}(v_i') = |N_{G'}(v_i')| = |[N_G[v_i]]^c| = |V - N_G[v_i]| = p - |N_G[v_i]| = p - (deg_G(v_i) + 1) = p - 1 - deg_G(v_i)$.

(iv) By definition, the graph $AD(\sigma G)$ is the graph obtained by adding k vertices v_i' $(1 \leq i \leq k)$ to the graph G. Hence, the number of vertices in $AD(\sigma G)$ is $p' = p + k$.

(v) The number of edges in $AD(\sigma G) =$ The number of edges in $G +$ the number of edges added after the anti-duplication of the k vertices in G. That is, $q' = q + \sum_{i=1}^{k} deg_{G'}(v_i')$. By (ii), $deg_{G'}(v_i') = p - 1 - deg_G(v_i)$, $1 \leq i \leq k$. Hence, $q' = q + \sum_{i=1}^{k} (p - 1 - deg_G(v_i)) = q + kp - k - \sum_{i=1}^{k} deg_G(v_i) = q + k(p-1) - \sum_{i=1}^{k} deg_G(v_i)$.

Hence the theorem.

Corollary 3.6. *Let G be a graph and let $v \in V(G)$. Let $G' = AD(vG)$. Then for $u \neq v$, $deg_{G'}(u)$ is either $deg_G(u)$ or $deg_G(u) + 1$.*

Proof. Let G be a graph and let $v \in V(G)$. Let $G' = AD(vG)$ and let $u \neq v$. By Theorem 3.5. (ii), we have, $deg_{G'}(u) = 1 +$ number of vertices in $V - v$ which are adjacent to u in G. If u is adjacent to v in G, then $deg_{G'}(u) = 1 + deg_G(u) - 1 = deg_G(u)$ and if u is not adjacent to v in G, then $deg_{G'}(u) = 1 + deg_G(u)$. Hence the theorem.

Result 3.7. *Let G be a graph and let $\sigma \subseteq V(G)$. Let $G' = AD(\sigma G)$. Then for $v \in \sigma$, $deg_{G[\sigma]}(v) = deg_{G'[\sigma]}(v)$.*

Proof. Let G be a graph and let $\sigma \subseteq V(G)$. Let v be a vertex in σ. By the definition of anti-duplication, we have, $G[\sigma] = G'[\sigma]$. Hence, $deg_{G[\sigma]}(v) = deg_{G'[\sigma]}(v)$.

Definition 3.8. *Let G be a graph and let $\sigma \subseteq V$ be such that $|\sigma| = k$. The* ***k-vertex anti-duplication switching*** *σ of G is the switching of the graph $AD(\sigma G)$ by σ. The resultant graph is denoted by $AD(\sigma G)^{\sigma}$.*

Theorem 3.9. *Let G be a graph and let $\sigma \subseteq V$ be a k-vertex anti-duplication switching of G. Let G' be the graph $AD(\sigma G)^{\sigma}$. Then for $v \in \sigma$, $deg_{G'}(v) = p - k + 3\, deg_{G[\sigma]}(v) - deg_G(v) + 1$.*

Proof. Let G be a graph and let $\sigma \subseteq V$ be the k-vertex anti-duplication switching of G. Let v be any vertex in σ. By definition of switching, in $AD(\sigma G)^{\sigma}$, the vertex v is adjacent to the vertices adjacent to v in $AD(\sigma G)[\sigma]$ and the vertices of $AD(\sigma G)[V - \sigma]$ that are non-adjacent to v in $AD(\sigma G)$. ___(1)

By Result 3.7, the number of vertices adjacent to v in $AD(\sigma G)[\sigma]$ is equal to $deg_{G[\sigma]}(v)$.

___(2)

The vertices of $AD(\sigma G)[V - \sigma]$ that are non-adjacent to v in $AD(\sigma G)$ are the vertices of $G[V - \sigma]$ that are non-adjacent to v in G and the anti-duplication vertices that are non-adjacent to v in $AD(\sigma G)$.
___(3)

Now, the number of vertices of $G[V - \sigma]$ that are non-adjacent to v in G is equal to the number of vertices in $G[V - \sigma]$ −number of vertices of $G[V - \sigma]$ that are adjacent to v in G.
___(4)

The number of vertices in $G[V - \sigma]$ is equal to $p - k$. ___(5)

The number of vertices of $G[V - \sigma]$ that are adjacent to v in G is equal to the number of vertices adjacent to v in G −the number of vertices adjacent to v in $G[\sigma] = deg_G(v) - deg_{G[\sigma]}(v)$.
___(6)

Hence, from equation (4), (5) and (6), we have, the number of vertices of $G[V - \sigma]$ that are non-adjacent to v in $G = p - k - (deg_G(v) - deg_{G[\sigma]}(v)) = p - k - deg_G(v) + deg_{G[\sigma]}(v)$.

___(7)

Now, the number of anti-duplication vertices that are non-adjacent to v in $AD(\sigma G)$ is equal to the number of vertices adjacent to v in $G[\sigma] + 1 = deg_{G[\sigma]}(v) + 1$. ___(8)

Hence, from (3), (7) and (8), the number of vertices of $AD(\sigma G)[V - \sigma]$ that are non-adjacent to v in $AD(\sigma G) = p - k - deg_G(v) + deg_{G[\sigma]}(v) + deg_{G[\sigma]}(v) + 1 = p - k - deg_G(v) + 2\, deg_{G[\sigma]}(v) + 1$.
___(9)

Thus, from (1), (2) and (9), we have, the number of vertices adjacent to v in $G' = deg_{G[\sigma]}(v) + p - k - deg_G(v) + 2\, deg_{G[\sigma]}(v) + 1$. That is, $deg_{G'}(v) = p - k + 3\, deg_{G[\sigma]}(v) - deg_G(v) + 1$.

Definition 3.10. *Let G be a graph and let $\sigma \subseteq V$ be a k-vertex anti-duplication switching of the graph G. Then σ is called as* ***k-vertex anti-duplication self switching*** *of G if $AD(\sigma G)^{\sigma} \cong AD(\sigma G)$ where $AD(\sigma G)$ is the graph obtained after the anti-duplication of the k vertices in G and $AD(\sigma G)^{\sigma}$ is the* *switching graph of $AD(\sigma G)$ by σ.*

Example 3.11. *Consider the graph G given in figure 2.3. Let $\sigma = \{v_2, v_4\}$. Here, $p = 4$ and $k = 2$.*

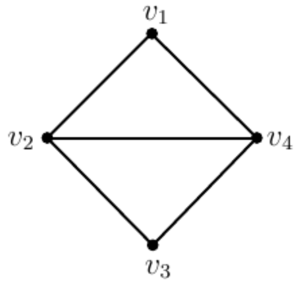

Fig 2.3 G

The graphs $AD(\sigma G)$ and $AD(\sigma G)^{\sigma}$ are given in figure 2.4 and figure 2.5, respectively. Clearly, $AD(\sigma G) \cong AD(\sigma G)^{\sigma}$. Hence, σ is a 2-vertex anti-duplication self switching of G.

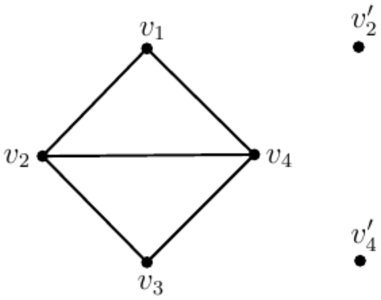

Fig 2.4 $AD(\sigma G)$

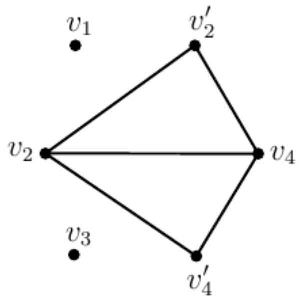

Fig 2.5 $AD(\sigma G)^{\sigma}$

Result 3.12. *Let $G(V, E)$ be a graph with p vertices and let $\sigma \subseteq V$ be a k-vertex anti-duplication self switching of G. Let G' be $AD(\sigma G)(V', E')$. Then the number of edges between the vertices of σ and $V' - \sigma$ in G' is $\frac{kp}{2}$.*

Proof. Let σ be a k-vertex anti-duplication self switching of G and let $G' = AD(\sigma G)$. By the definition of anti-duplication self switching, σ is a k-vertex self switching of $AD(\sigma G)$ and hence $AD(\sigma G) \cong AD(\sigma G)^{\sigma}$. By Theorem 3.5. (iv), G' has $p' = p + k$ vertices and by Theorem 2.5, the number of edges between the vertices of σ and $V' - \sigma$ in $AD(\sigma G)$ and in $AD(\sigma G)^{\sigma}$ is $\frac{k(p'-k)}{2} = \frac{k(p+k-k)}{2} = \frac{kp}{2}$.

Result 3.13. *Let $G(V, E)$ be a graph and let $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$ be a k-vertex anti-duplication self switching of G. Then the number of edges between the vertices of σ in $AD(\sigma G)^{\sigma}$ is $\frac{1}{2}\sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$.*

Proof. Let G be a graph and let $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$ be a k-vertex anti-duplication self switching of G. Clearly, the number of edges between the vertices of σ in $AD(\sigma G)^{\sigma}$, $AD(\sigma G)$ and G are equal. Hence, the number of edges between the vertices of σ in $AD(\sigma G)^{\sigma} = \frac{1}{2}\sum_{i=1}^{k} \deg_{G[\sigma]}(v_i)$.

Theorem 3.14. *Let $G(V, E)$ be a graph with p vertices and let $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V$ be a k-vertex anti-duplication self switching of*

G. *Then the number of edges between the vertices of* σ *and* $V - \sigma$ *in* G *is* $\frac{kp}{2} - k(k-1) + \sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$.

Proof. Let σ be a k-vertex anti-duplication self switching of G and let $G' = AD(\sigma G)(V', E')$. Let $\sigma' = \{v_1', v_2', \dots, v_k'\}$ where v_1', v_2', ..., v_k' are the anti-duplication vertices of v_1, v_2, ..., v_k, respectively. By the definition of anti-duplication, the number of edges between the vertices of σ and $V - \sigma$ in G and the number of edges between the vertices of σ and $V - \sigma$ in G' are equal. Since $V'(G') = V(G) \cup \sigma'$, the number of edges between the vertices of σ and $V - \sigma$ in G' = the number of edges between the vertices of σ and $V' - \sigma$ in G' − the number of edges between the vertices of σ and σ' in G'.

___(1)

By Result 3.12, the number of edges between the vertices of σ and $V' - \sigma$ in $G' = \frac{kp}{2}$.

___(2)

The number of edges between the vertices of σ and σ' in $G' = \sum_{i=1}^{k}($ the number of vertices that are non-adjacent to v_i in σ of G). The number of vertices that are non-adjacent to v_i in σ of $G = k - deg_{G[\sigma]}(v_i) - 1$. Hence, the number of edges between the vertices of σ and σ' in $G' = \sum_{i=1}^{k} (k - deg_{G[\sigma]}(v_i) - 1) = k^2 - \sum_{i=1}^{k} deg_{G[\sigma]}(v_i) - k$. ___(3)

Thus from equations (1), (2) and (3), the number of edges between the vertices of σ and $V - \sigma$ in $G = \frac{kp}{2} - (k^2 - \sum_{i=1}^{k} \deg_{G[\sigma]}(v_i) - k) = \frac{kp}{2} - k(k-1) + \sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$.

Theorem 3.15. *Let* G *be a graph and let* $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$ *be a* k*-vertex anti-duplication self switching of* G. *Then* $\sum_{i=1}^{k} deg_G(v_i) = \frac{kp}{2} - k(k-1) + 2\sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$.

Proof. Let σ be a k-vertex anti-duplication self switching of G and let $G' = AD(\sigma G)$. Now, $\sum_{i=1}^{k} deg_G(v_i)$ = The number of edges between the vertices of σ and $V - \sigma$ in $G + 2$(the number of edges between the vertices of σ in G). By Theorem 3.14, the number of edges between the vertices of σ and $V - \sigma$ in $G = \frac{kp}{2} - k(k-1) + \sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$. Also, the number of edges between the vertices of σ in $G = \frac{1}{2}\sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$. Hence, $\sum_{i=1}^{k} deg_G(v_i) = \frac{kp}{2} - k(k-1) + \sum_{i=1}^{k} deg_{G[\sigma]}(v_i) + 2\left[\frac{1}{2}\left(\sum_{i=1}^{k} deg_{G[\sigma]}(v_i)\right)\right] = \frac{kp}{2} - k(k-1) + 2\sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$. Hence the theorem.

Remark 3.16. *The converse of the above theorem need not be true. For example, consider the graph G given in figure 2.6. Let* $\sigma = \{v_1, v_2, v_4\}$. *Then* $k = 3$ *and* $p = 6$. *Now,* $deg_G(v_1) + deg_G(v_2) + deg_G(v_4) = 2 + 2 + 3 = 7$. *Also,* $\frac{kp}{2} - k(k-1) + 2\sum_{i=1}^{k} deg_{G[\sigma]}(v_i) = \frac{(3)(6)}{2} - 3(3-1) + 2(1 + 1 + 0) = 9 - 6 + 4 = 7$.

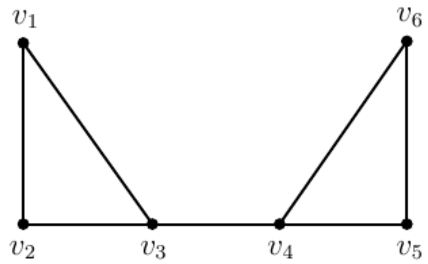

Fig 2.6 G

The graphs $AD(\sigma G)$ and $AD(\sigma G)^{\sigma}$ are given in figure 2.7 and 2.8, respectively. Clearly, $AD(\sigma G) \ncong AD(\sigma G)^{\sigma}$. Hence, σ is not a k-vertex anti-duplication self switching of G.

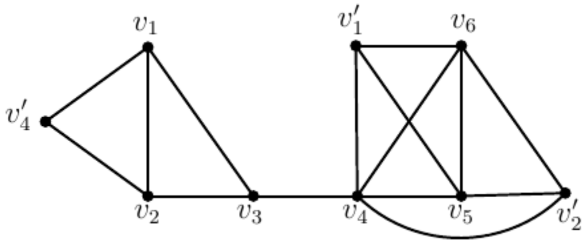

Fig 2.7 $AD(\sigma G)$

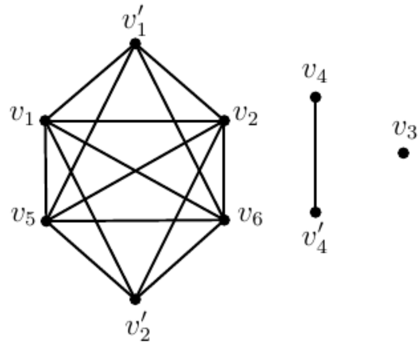

Fig 2.8 $AD(\sigma G)^{\sigma}$

Theorem 3.17. *Let G be a graph with p vertices and $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$ be a k-vertex anti-duplication self switching of G. Let $G' = AD(\sigma G)$. Then $\sum_{i=1}^{k} deg_{G'}(v_i) = \frac{kp}{2} + \sum_{i=1}^{k} deg_{G'[\sigma]}(v_i)$.*

Proof. Let G be a graph with p vertices and $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$ be a k-vertex anti-duplication self switching of G. Let $G' = AD(\sigma G)$ with $p + k$ vertices. Then by the definition of anti-duplication, σ is a k-vertex self switching of G'. By Theorem 2.6, we have, $\sum_{i=1}^{k} deg_{G'}(v_i) = \frac{k(p+k-k)}{2} + 2$(the number of edges between the vertices of σ in G') $= \frac{kp}{2} + 2\left[\frac{1}{2}\left(\sum_{i=1}^{k} deg_{G'[\sigma]}(v_i)\right)\right] = \frac{kp}{2} + \sum_{i=1}^{k} deg_{G'[\sigma]}(v_i)$.

Theorem 3.18. *Let G be a graph with p vertices and $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$ be a k-vertex anti-duplication self switching of G. Let $G' = AD(\sigma G)$. Then $\sum_{i=1}^{k} deg_{G'}(v_i) = \frac{kp}{2} + \sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$.*

Proof. Let G be a graph with p vertices and $\sigma = \{v_1, v_2, \dots, v_k\} \subseteq V(G)$ be a k-vertex anti-duplication self switching of G. Let $G' = AD(\sigma G)$ with $p + k$ vertices. By Theorem 3.17, we have, $\sum_{i=1}^{k} deg_{G'}(v_i) = \frac{kp}{2} + \sum_{i=1}^{k} deg_{G'[\sigma]}(v_i)$. By Theorem 3.7, $deg_{G[\sigma]}(v_i) = deg_{G'[\sigma]}(v_i)$, $1 \leq i \leq k$. Hence, $\sum_{i=1}^{k} deg_{G'}(v_i) = \frac{kp}{2} + \sum_{i=1}^{k} deg_{G[\sigma]}(v_i)$.

Remark 3.19. *The converse of the above theorem need not be true. For example, consider the graph given in figure 2.6. Let $\sigma = \{v_1, v_2, v_4\}$. Then $k = 3$ and $p = 6$. The graph $AD(\sigma G)$ is given in figure 2.7. Now, $\sum_{i=1}^{k} deg_{G'}(v_i) = 3 + 3 + 5 = 11$. Also,*

$\frac{kp}{2} + \sum_{i=1}^{k} deg_{G'[\sigma]}(v_i) = \frac{kp}{2} + \sum_{i=1}^{k} deg_{G[\sigma]}(v_i) = \frac{(3)(6)}{2} + (1 + 1 + 0) = 11$. *The graph* $AD(\sigma G)^\sigma$ *is given in figure 2.8. Clearly,* $AD(\sigma G) \ncong AD(\sigma G)^\sigma$. *Hence,* σ *is not a* k*-vertex anti-duplication self switching of* G.

Remark 3.20. *An odd order graph* G *has no odd* k*-vertex anti-duplication self switching.*

Proof. Let G be a graph with odd order p and let σ be a k-vertex anti-duplication self switching of G where k is odd. Let $G' = AD(\sigma G)$. By Theorem 3.5. (iv), G' has $p' = p + k$ vertices. Since both p and k are odd, G' has even number of vertices p'. By Theorem 2.7, G' has no odd k-vertex self switching. Hence, G has no odd k-vertex anti-duplication self switching.

Conclusion

The concept of k-vertex anti-duplication self switching has been proposed and its associated properties have been analyzed in this paper.

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