International Journal of
INTELLIGENT SYSTEMS AND APPLICATIONS IN ENGINEERING
ISSN:2147-6799 www.ijisae.org Original Research Paper

Total Dominator Color Class Total Dominating Sets in Necklace Graph, Hurdle Graph and F-Tree Graph

Dr. A. Vijayalekshmi[1], P. Jaspin Ida[2]

Submitted: 12/04/2024 **Revised:** 29/05/2024 **Accepted:** 10/06/2024

Abstract: Let $G = (V, E)$ be a finite, undirected and connected graph with minimum degree at least one. A proper coloring $\mathcal{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in V(G). A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$. Here we obtain $\gamma_\chi^{td}(G)$ for Necklace graph, Hurdle graph and F-tree graph.

Keywords: *Chromatic number, Domination number, Color class dominating set, Dominator color class dominating set, Total dominator color class total domination number.*

AMS Subject Classification: 05C15, 05C69.

1. Introduction

All graphs considered in this paper are finite, undirected graphs with minimum degree at least one and we follow standard definitions of graph theory as found in [9]. Let G = (V,E) be a connected graph with no isolated vertices. The open neighborhood N(v) of a vertex v ∈ V(G) consists of the set of all vertices adjacent to v. The closed neighborhood of v is N[v] = N(v)∪{v}. For a set S ⊆ V, the open neighborhood N(S) is defined to be $\cup_{v\in S}N(v)$, and the closed neighborhood of S is N[S] = N(S)∪S. For any set H of vertices of G, the induced subgraph ⟨H⟩ is the maximal subgraph of G with vertex set H. A subset S of V is called a dominating set if every vertex in V−S is adjacent to some vertex in S.

1Associate Professor & Head, Department of Mathematics, S.T.Hindu College, Nagercoil-629002, Tamilnadu, India, Email: vijimath.a@gmail.com

(Affliated to Manonmaniam Sundaranar University, Tirunelveli-627012, Tamilnadu, India)

2Research Scholar, Reg.No:20123152092018, Department of Mathematics, S.T.Hindu College, Nagercoil-629002, Tamilnadu, India, Email: jaspinida13@gmail.com

(Affliated to Manonmaniam Sundaranar University, Tirunelveli-627012, Tamilnadu, India)

A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$ is the minimum cardinality taken over all minimal dominating sets of G. A γ-set of G is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(G)$. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class The total dominator chromatic number is denoted by $\chi_{td}(G)$. This notion was introduced by A.Vijayalekshmi et al [10]. A color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that every color classes in $\mathcal{C}$ is dominated by a vertex in G. A color class dominating set $\mathcal{C}$ is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_\chi(G)$. This notion was introduced by A.Vijayalekshmi et al [4].

A dominator color class dominating set of G is a proper coloring $\mathcal{C}$ of G with the extra property that each vertex v in G is dominated by a color class $\mathcal{C}_I \in \mathcal{C}$ and each color class $\mathcal{C}_I \in \mathcal{C}$ is dominated by a vertex in G. The dominator color class domination number of G is the minimum cardinality taken over all dominator color class dominating sets in G and is denoted by γ_χ^{d} (G). This notion was introduced by A.Vijayalekshmi et al [5]. A proper coloring $\mathcal{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in V(G). A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by γ_χ^{td}(G). This notion was introduced by A.Vijayalekshmi et al [6]. The join $G_1 + G_2$ of graphs G_1 and G_2 with disjoint vertex set V_1 and V_2 and edge sets E_1 and E_2 respectively, is the graph union $G_1 \cup G_2$ together with each vertex in V_1 is adjacent to every vertices in V_2. A necklace graph denoted by Ne_n is a cubic halin graph obtained by joining a cycle with all vertices of degree 1 of a caterpillar (also called a comb) having n vertices of degree 3 and n+2 vertices of degree 1. A hurdle graph formed from a Path graph(P_n) by adding pendant edge to each internal vertex of the path, resulting in n-2 "hurdles". A F-tree $F(P_n)$ is a graph obtained from path n≥3 vertices by appending two pendant edges one to an end vertex and the other to a vertex adjacent to an end vertex.

We shall use the following observation from [6]

Theorem:A

Let G be P_n or $C_n. Then\ for\ n \geq 5,$

$$\gamma_\chi^{td}(P_n) = \gamma_\chi^{td}(C_n) = \begin{cases} \left\lfloor \frac{2n}{3} \right\rfloor & if\ n \equiv 0(mod3) \\ 2\left\lfloor \frac{n+2}{3} \right\rfloor & if\ n \not\equiv 0(mod3) \end{cases}$$

2.Main Results

Theorem:2.1

Let Ne_n be a necklace graph of order n. Then

$$\gamma_\chi^{td}(Ne_n) = \begin{cases} \left\lfloor \frac{4(n+1)}{3} \right\rfloor + 1 & \text{if } n \equiv 0(mod3) \\ \left\lfloor \frac{4(n+1)}{3} \right\rfloor + 2 & \text{if } n \equiv 1(mod3) \\ \left\lceil \frac{4(n+1)}{3} \right\rceil & \text{if } n \equiv 2(mod3) \end{cases}$$

Proof:

Let $V(Ne_n) = \{u, v, u_i, v_i / 1 \leq i \leq n\}$ and deg(u)=deg(v)=3 and deg(u_i)=deg(v_i)=2, for every i=1,2,3,…,n.

We consider 3 cases.

Case:1 When $n \equiv 0(mod3)$

Assign distinct colors say 1,2, $\left\lfloor \frac{4(n+2)}{3} \right\rfloor + 1$ and $\left\lfloor \frac{4(n+2)}{3} \right\rfloor + 2$ to the vertices {u},{u_1,v},{v_{n-1},u_n} and {v_n} respectively. Assign distinct colors say 2i+1 and 2i+2 for 1≤ i ≤ $\left(\frac{n}{3} - 1\right)$ to the vertices say {u_{3i-1},u_{3i+1}} and {u_{3i}} and for $\left(\frac{n}{3} + 1\right) \leq i \leq \left(\frac{n}{3} + 2\right)$ to the vertices {v_{3i-1}, v_{3i+1}} and {v_{3i}}respectively, we obtain a γ_χ^{td}-coloring of Ne_n.

So $\gamma_\chi^{td}(Ne_n) = \left\lfloor \frac{4(n+1)}{3} \right\rfloor$+1

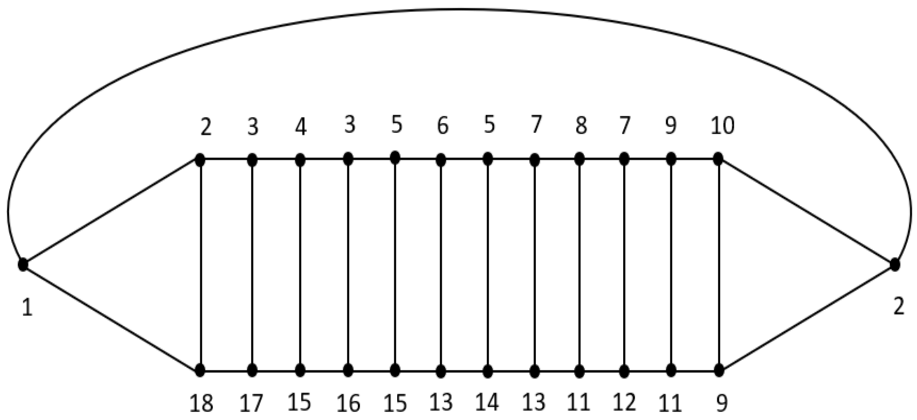

Figure 1 $\gamma_\chi^{td}(Ne_{12}) = 18$

Case:2 When $n \equiv 1(\text{mod}3)$

Assign same coloring of 1 and 2 to the same vertices in case (1). Also we assign distinct colors say 2i+1 and 2i+2 to the vertices $\{u_{3i-1}, u_{3i+1}\}$ and $\{u_{3i}\}$ for $1 \leq i \leq \left(\frac{n}{3}\right)$ and to the vertices $\{v_{3i-1}, v_{3i+1}\}$ and $\{v_{3i}\}$ for $\left(\frac{n}{3}+1\right) \leq i \leq \left(\frac{n}{3}+2\right)$, we obtain a γ_{χ}^{td}- coloring of Ne_n. Thus $\gamma_{\chi}^{td}(Ne_n) = \left\lfloor \frac{4(n+1)}{3} \right\rfloor + 2$

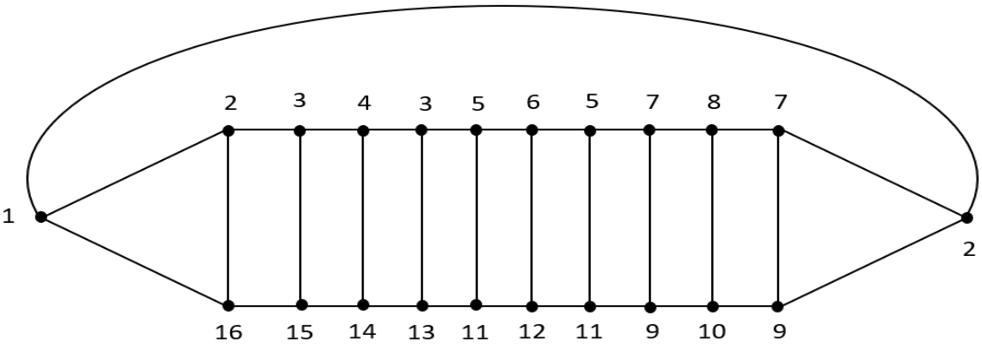

Figure:2 $\gamma_{\chi}^{td}(Ne_{10}) = 16$

Case:3 When $n \equiv 2(\text{mod}3)$

Assign distinct colors say 1,2, $\left\lceil \frac{4(n+1)}{3} \right\rceil - 1$ and $\left\lceil \frac{4(n+1)}{3} \right\rceil$ to the vertices say $\{u\}, \{u_1, v_1\}, \{u_n, v_n\}$ and $\{v\}$ respectively. Since $n-1 \equiv 1(\text{mod}3)$ and as in Case (2), $\gamma_{\chi}^{td}(Ne_n) = \gamma_{\chi}^{td}(Ne_{n-1}) + 2 = \frac{4(n+1)}{3}$

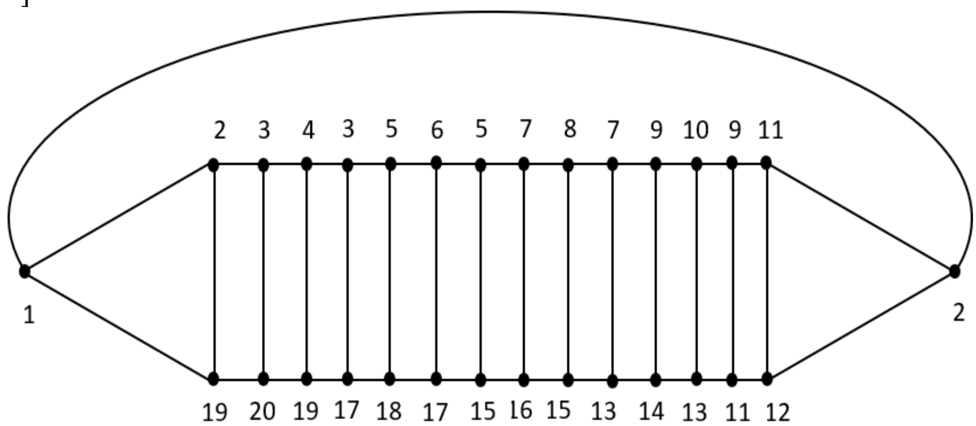

Figure:3 $\gamma_{\chi}^{td}(Ne_{14}) = 20$

Theorem:2.2

For a Hurdle graph, $\gamma_{\chi}^{td}(Hd_n) = 2n-4, n \geq 3$

Proof:

Let $V(Hd_n) = \{u_i, v_j \,/\, 1 \leq i \leq n, 1 \leq j \leq n-2\}$ with $deg\ u_1 = deg\ v_n = 2, deg\ u_i = 3, 2 \leq i \leq n-1$ *and* $\deg v_j = 1$, $1 \leq j \leq n-2$. Assign distinct colors say 2i($1 \leq i \leq n-2$) and 2j-1 ($2 \leq j \leq n-3$) to the vertices say $\{u_{i+1}\}$ and $\{v_j\}$ respectively. Also assign distinct colors say 1 and 2n-4 to the vertices say $\{u_1, v_1\}$ and $\{u_n, v_{n-2}\}$, we obtain a $\gamma_{\chi}^{td} - coloring\ of\ Hd_n$. So $\gamma_{\chi}^{td}(Hd_n) = 2n-4, n \geq 3$

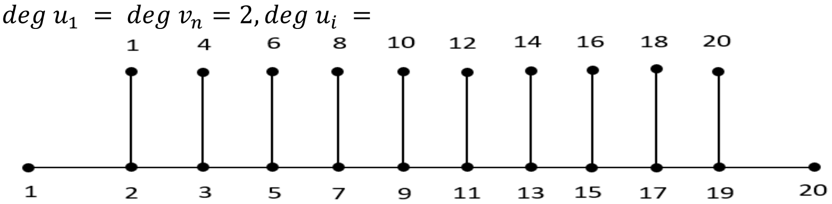

Figure:4 $\gamma_{\chi}^{td}(Hd_{12}) = 20$

Theorem:2.3

For a F-tree graph, $\gamma_\chi^{td}(F(P_n)) = \begin{cases} \left\lfloor \frac{2n}{3} \right\rfloor + 2 & if\ n \equiv 0(mod3) \\ \left\lfloor \frac{2(n+2)}{3} \right\rfloor + 2 & if\ n \not\equiv 0(mod3) \end{cases}$

Proof:

Let $V(F(P_n)) = \{u_i, v_1, v_2 / 1 \le i \le n\}$ $with\ V(P_n) = \{u_i / 1 \le i \le n\}$ and $v_1\ and\ v_2$ are pendant vertices adjacent to u_{n-1} and u_n respectively.

We consider 2 cases.

Case :1 When n≡ 0(mod 3)

As in the same $\gamma_\chi^{td}(P_{n-3})$ together with four distinct colors say $\left\lfloor \frac{2n}{3} \right\rfloor - 1, \left\lfloor \frac{2n}{3} \right\rfloor, \left\lfloor \frac{2n}{3} \right\rfloor + 1\ and\ \left\lfloor \frac{2n}{3} \right\rfloor + 2$ to the vertices say $\{u_{n-2}, v_n\}$, $\{v_{n-2}\}$, $\{u_n\}$ and $\{v_2\}$ respectively, we obtain a γ_χ^{td}-coloring of $F(P_n)$. Thus $\gamma_\chi^{td}(F(P_n)) = \left\lfloor \frac{2n}{3} \right\rfloor + 2$

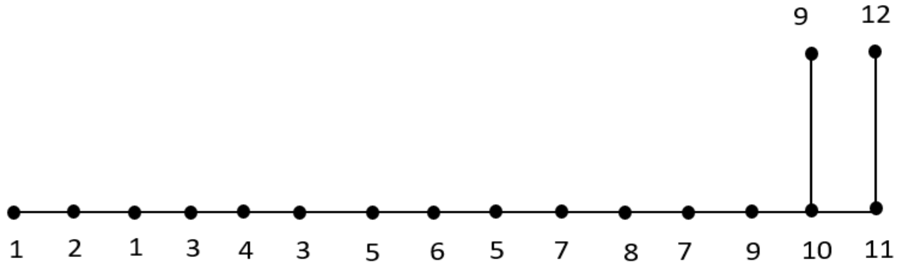

Figure:5 $\gamma_\chi^{td}(F(P(15))) = 12$

Case:2 When $n \not\equiv 0(mod3)$

We consider 2 subcases.

Subcase:2.1 When $n \equiv 1(mod3)$

Since n-4 $\equiv 1(mod3)$. As in the same γ_χ^{td}-coloring of P_{n-4} together with six distinct colors say $\left\lfloor \frac{2(n+2)}{3} \right\rfloor - 3$, $\left\lfloor \frac{2(n+2)}{3} \right\rfloor$-2, $\left\lfloor \frac{2(n+2)}{3} \right\rfloor - 1, \left\lfloor \frac{2(n+2)}{3} \right\rfloor, \left\lfloor \frac{2(n+2)}{3} \right\rfloor + 1\ and\ \left\lfloor \frac{2(n+2)}{3} \right\rfloor + 2$ to the vertices say $\{u_{n-3}\}$, $\{u_{n-2}\}$, $\{u_{n-1}\}$, $\{v_1\}$, $\{u_n\}$ and $\{v_2\}$ respectively, we get a γ_χ^{td}-coloring of $F(P_n)$. Thus $\gamma_\chi^{td}(F(P_n)) = \left\lfloor \frac{2(n+2)}{3} \right\rfloor + 2$

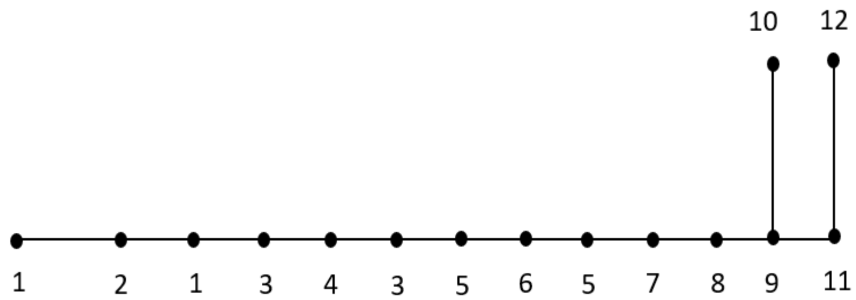

Figure:6 $\gamma_\chi^{td}(F(P(13))) = 12$

Subcase:2.2 When $n \equiv 2(mod3)$

As in the same γ_χ^{td}-coloring of P_{n-2} together with four distinct colors say $\left\lfloor \frac{2(n+2)}{3} \right\rfloor - 1, \left\lfloor \frac{2(n+2)}{3} \right\rfloor, \left\lfloor \frac{2(n+2)}{3} \right\rfloor + 1\ and\ \left\lfloor \frac{2(n+2)}{3} \right\rfloor + 2$ to the vertices say $\{v_2\}$, $\{u_n\}$, $\{v_1\}$ and $\{u_{n-1}\}$ respectively, we get a γ_χ^{td}-coloring of $F(P_n)$. Thus $\gamma_\chi^{td}(F(P_n)) = \left\lfloor \frac{2(n+2)}{3} \right\rfloor + 2$

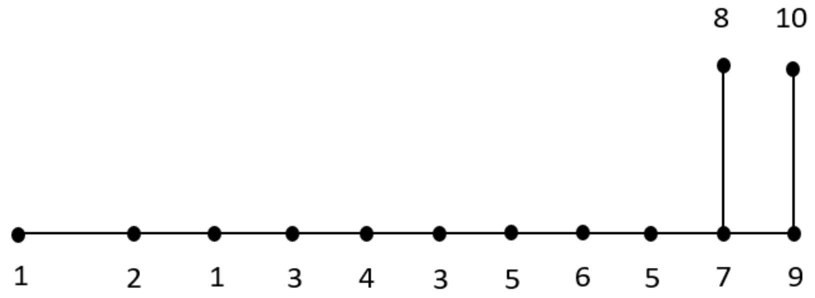

Figure:7 $\gamma_{\chi}^{td}(F(P(11))) = 10$

References

[1] A. Vijayalekshmi, Total Dominator Colorings in Graphs, International Journal of Advancements in Research & Technology, 1(4) (2012), 1-10.

[2] A.Vijayalekshmi and S.Anusha, Total Dominator Chromatic Number on Various Classes of Graphs, International Journal of Scientific Research and Reviews, 8(4)(2019), 32-40.

[3] A.Vijayalekshmi and J. Virjin Alangara Sheeba, Total Dominator Chromatic Number of Paths Cycles and Ladder graphs, International Journal of Contemporary Mathematical Sciences, 13(5)(2018), 199–204.

[4] Harishchandra Patel, "Impedance Control in HDI and Substrate-Like PCBs for AI Hardware Applications" (2024). Journal of Electrical Systems, 20(11s), 5109-5115.

[5] A.Vijayalekshmi and A.E.Prabha, Introduction of color class dominating sets in Graphs, Malaya Journal of Matematik, 8(4)(2020), 2186–2189.

[6] A.Vijayalekshmi and P. Niju, An Introduction of Dominator color class dominating sets in Graphs, Malaya Journal of Matematik, 9(1)(2021), 1015–1018.

[7] A.Vijayalekshmi and S. Abisha, An introduction of total dominator color class total dominating sets in Graphs, Malaya Journal of Matematik, vol.9, No.1, 1225-1228, 2021.

[8] A.Vijayalekshmi and S. Anusha, Dominator Chromatic Number on Various Classes of Graphs, International Journal of Scientific Research and Reviews, 9(3)(2020), 91– 101.

[9] A.Vijayalekshmi and A.E.Prabha, Color Class Dominating Sets in various classes of graphs, Malaya Journal of Matematik, Vol. 9, No.1, pp. 195-198, 2021.

[10] F. Harrary, Graph theory, Addition-Wesley Reading Mass,1969.

[11] M.I.Jinnah and A.Vijayalekshmi, Total Dominator Colorings in Graphs, Ph.D Thesis, University of Kerala, 2010.

[12] A.E.Prabha and A.Vijayalekshmi, Color Class Dominating Sets in ladder and grid graphs, Malaya Journal of Matematik, Vol. 9, No.1, pp. 993-995, 2021.

[13] A.Vijayalekshmi and S.G.Vidhya, Color class dominating sets in path and cycle related Graphs, Malaya Journal of Matematik, vol.9, No.1, 1237-1240, 2021.

[14] A.Vijayalekshmi and S. Abisha, Total dominator color class total dominating sets in Dutch windmill graph and coconut tree graph, Malaya Journal of Matematik, vol.9, No.1, 1229-1232, 2021.

[15] A.Vijayalekshmi and S.Abisha, Total dominator color class total dominating sets in ladder and mobius ladder graph, Malaya Journal of Matematik, vol.9, No.1, 1233-1236, 2021.

[16] A.Vijayalekshmi and P.Niju, Dominator color class dominating sets on firecracker, gear and flower graphs, Malaya Journal of Matematik, vol.9, No.1, 1043-1046, 2021.

[17] A.Vijayalekshmi and S.Anusha, Total dominator chromatic number of $P_m \times C_n$, Malaya Journal of Matematik, vol.8, No.4, 1469-1472, 2020.

[18] A.Vijayalekshmi and S.G.Vidhya, Dominator color class dominating sets in Grid graphs, Journal of computational Analysis and Applications, vol.33, No.8, 385-389, 2024.

[19] R.Ponraj and S.Prabhu, Pair Mean Cordial Labelling of Hurdle, Key, Lotus and Necklace Graphs, Journal of Mathematics and its Applications, vol.18, Issue.4, 2795-2804, 2024.

[20] Terasa W. Haynes, Stephen T.Hedetniemi, Peter Slater, Domination in Graphs, Advanced Topics, New York, 1998.

[21] Terasa W. Haynes, Stephen T. Hedetniemi, Peter Slater, Domination in Graphs, Marcel Dekker, New York, 1998.