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Total Dominator Colour Class Total Dominating Sets in Diagonal Ladder Graph, Pentagonal Circular Ladder Graph and Pencil Graph

Dr.A.Vijayalekshmi[1], A. Giftlin Beatrice Adulshiya [2]

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Abstract: *Let $G = (V, E)$ be a finite, undirected and connected graph. A proper coloring $\mathcal{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in $V(G)$. A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating Set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_{\chi td}(G)$. Here we obtain total dominator color class total domination number of, Diagonal Ladder graph, Pentagonal Circular Ladder graph and Pencil graph.*

Keywords: *Chromatic number, Total Domination number, Dominator Color Class Domination number, Total Dominator Color Class Total Domination number.*

1. Introduction

All graphs considered in this paper are finite, undirected and connected graphs without loops and multiple edges. We follow standard definitions of graph theory as found in [19].

A subset *S* of *V* is called a total dominating set if every vertex in $V(G)$ is adjacent to some vertex in *S*. A dominating set *S* is called a minimal total dominating set if no proper subset of *S* is a total dominating set of *G*. The total domination number $\gamma_t(G)$ is the minimum cardinality taken over all minimal total dominating sets of *G*. A proper coloring of G is an assignment of colors to the vertices of *G* such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of *G* is called chromatic number of *G* and is denoted by $\chi(G)$. A total dominator coloring of *G* is a proper coloring of *G* with the extra property that every vertex in *G* properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(G)$. This notion was introduced by A.Vijayalekshmi et al [20]. A color class dominating set of *G* is a proper coloring $\mathcal{C}$ of *G* with the extra property that every color classes in $\mathcal{C}$ is dominated by a vertex in *G*. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathcal{C}$ is a color class dominating set of *G*. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of *G* and is denoted by $\gamma_\chi(G)$. This notion was introduced by A.Vijayalekshmi et al[1].

A dominator color class dominating set of *G* is a proper coloring $\mathcal{C}$ of *G* with the extra property that each vertex *v* in *G* is dominated by a color class $\mathcal{C}_i \in \mathcal{C}$ and each color class $\mathcal{C}_i \in \mathcal{C}$ is dominated by a vertex in *G*. The dominator color class domination number of *G* is the minimum cardinality taken over all dominator color class dominating sets in *G* and is denoted by $\gamma_\chi^d(G)$. This notion was introduced by A.Vijayalekshmi et al [3]. A proper coloring $\mathcal{C}$ of *G* is said to be a Total Dominator Color Class Total Dominating set of *G* if each vertex properly dominates a color class in $\mathcal{C}$ and each color class in $\mathcal{C}$ is properly dominated by a vertex in $V(G)$. A Total Dominator Color Class Total Dominating set *D* of *G* is a minimal total dominator color class total dominating set if no proper subset of *D* is a total dominator color class total dominating set of *G*. The total dominator color class total domination number is the minimum cardinality taken over all

[1] Associate Professor, Department of Mathematics, S.T.Hindu College, Nagercoil-629002, Tamilnadu, India. Email: vijimath.a@gmail.com
ORCID ID : 0009-0006-5358-1170
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627012, Tamil Nadu, India)
[2] Research Scholar, Reg.No: 23113152092004, Department of Mathematics, S.T.Hindu College, Nagercoil-629002, Tamil Nadu, India. Email: giftlinadulshiya@gmail.com
ORCID ID : 0009-0003-5453-8201
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627012, Tamil Nadu, India)

minimal total dominator color class total dominating sets in G and is denoted by $\gamma_{\chi}^{td}(G)$. This notion was introduced by A.Vijayalekshmi et al [7].

The ladder graph L_n is a Cartesian product graph of P_2 and P_n where n ≥ 2.The Diagonal Ladder DL_n is the graph obtained from the ladder L_n by adding two diagonals to each rectangle. The Circular ladder graph CL_n is a Cartesian product graph of P_2 and C_n with vertex set $\{u_i, v_i: 1 \le i \le n\}$ and edge set $\{u_iv_i: 1 \le i \le n\} \cup \{v_iv_{i-1}, u_iu_{i-1}: 1 \le i \le n-1\}$ where n ≥ 3. The Pentagonal Circular Ladder graph PCL_n is a Circular ladder graph CL_n by adding one new vertex between each v_i for all $1 \le i \le n$. Pencil graph Pc_n is a graph with vertex set $V(Pc_n) = \{u, v, u_i, v_i: 1 \le i \le n\}$and edge set $E(Pc_n) = \{u_iu_{i+1}, v_iv_{i+1}: 1 \le i \le n-1\} \cup \{u_iv_i: 1 \le i \le n\} \cup \{uv, uu_1, uv_1, vu_n, vv_n\}$.

2. MAIN RESULTS

Theorem 2.1: Let DL_n be a Diagonal Laddar graph where $n \ge 3$. Then

$$\gamma_{\chi}^{td}(DL_n) = \begin{cases} \frac{4n}{3} & if\ n \equiv 0 (mod\ 3) \\ \frac{4(n+2)}{3} - 2 & if\ n \equiv 1 (mod\ 3) \\ \frac{4(n+1)}{3} & orelse \end{cases}$$

Proof

Let $V(DL_n) = \{u_i, v_i: 1 \le i \le n\}$ with $\deg(u_1) = \deg(u_2) = \deg(v_1) = \deg(v_2) = 2$ and $\deg(u_i) = \deg(v_i) = 4$ for all $2 \le i \le n-1$. We consider three cases.

Case 1: When $n \equiv 0(mod\ 3)$

Assign color 4i-3, 4i-2, 4i-1 and 4i to the vertices $\{u_{3i-2}, v_{3i}\}$, $\{u_{3i-1}\}$, $\{v_{3i-2}, u_{3i}\}$ and $\{v_{3i-1}\}$ respectively$\left(1 \le i \le \frac{n}{3}\right)$. We obtain γ_{χ}^{td}- coloring of DL_n. So $\gamma_{\chi}^{td}(DL_n) = \frac{4n}{3}$.

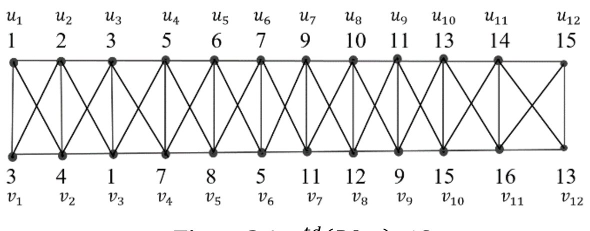

Figure 2.1. $\gamma_{\chi}^{td}(DL_{12})$=16

Case 2: When $n \equiv 1(mod\ 3)$

Since $n-1 \equiv 0(mod\ 2)$ by case 1, $\gamma_{\chi}^{td}(PCL_{n-1}) = \frac{4(n-1)}{3}$. Assign color $\{u_n\}$ and $\{v_n\}$ to the vertices $\frac{4(n+2)}{3}$ -3 and $\frac{4(n+2)}{3}$ -2 respectively we get γ_{χ}^{td}- coloring of DL_n. Thus $\gamma_{\chi}^{td}(DL_n) = \frac{4(n+2)}{3}$ -2.

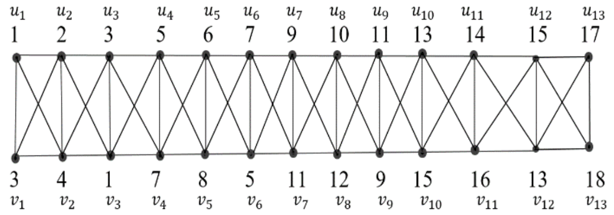

Figure 2.2. $\gamma_{\chi}^{td}(DL_{13})$=18

Case 3: When $n \equiv 2(mod\ 3)$

Since $n-1 \equiv 1(mod\ 3)$ by case 2, $\gamma_{\chi}^{td}(PCL_{n-2}) = \frac{4(n+1)}{3}$ -2 . Assign color $\{u_n\}$ and $\{v_n\}$ to the vertices $\frac{4(n+1)}{3}$ -1 and $\frac{4(n+1)}{3}$ respectively we get γ_{χ}^{td}- coloring of DL_n. Thus $\gamma_{\chi}^{td}(DL_n) = \frac{4(n+1)}{3}$.

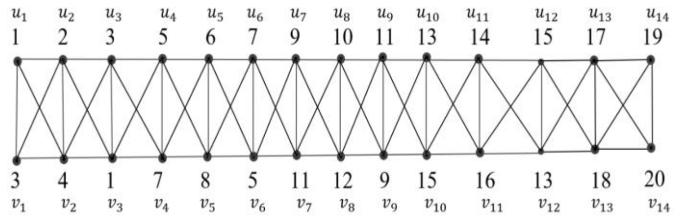

Figure 2.3. $\gamma_{\chi}^{td}(DL_{14})$=20

Theorem 2.2: Let PCL_n be a Pentagonal Circular Laddar graph where $n \ge 3$. Then $\gamma_{\chi}^{td}(PCL_n)$ = 2n.

Proof

Let $V(PCL_n) = \{u_i, v_i, w_i: 1 \le i \le n\}$ with $\deg(u_i) = \deg(v_i) = 3$ and $\deg(w_i) = 2$ for all $1 \le i \le n$. We consider two cases.

Case 1: When $n \equiv 0(mod\ 2)$

Assign color $i(1 \le i \le 4)$ to the vertices $\{w_n, u_1, w_1\}, \{v_1\}, \{u_2\}$and $\{v_2\}$ respectively. The vertices $\{w_{2i}, u_{2i+1}, w_{2i+1}\}, \{v_{2i+1}\}, \{u_{2i+2}\}$ and $\{v_{2i+2}\}$on assigning colors 4i+1, 4i+2, 4i+3 and 4i+4 respectively$\left(1 \le i \le \frac{n}{2} - 1\right)$ we obtain γ_{χ}^{td}- coloring of PCL_n. Thus $\gamma_{\chi}^{td}(PCL_n)$ = 2n.

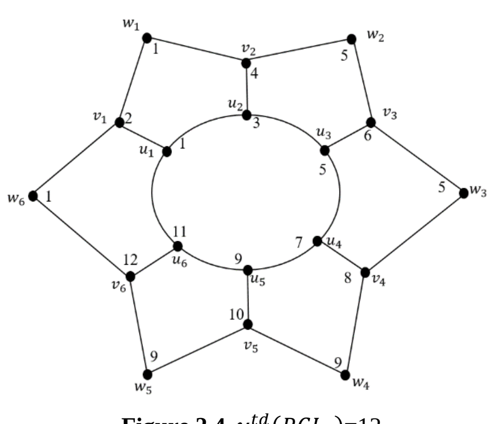

Figure 2.4. $\gamma_{\chi}^{td}(PCL_6)$=12

Case 2: When $n \equiv 1(mod\ 2)$

Since $n-1 \equiv 0(mod\ 2)$ by case 1, $\gamma_{\chi}^{td}(PCL_{n-1})$ = 2n-2. The remaining vertices $\{u_n, w_{n-1}\}$ and $\{v_n\}$ are assigned colors

2n-1 and 2n respectively to get γ_{χ}^{td}- coloring of PCL_n. So $\gamma_{\chi}^{td}(PCL_n)$ = 2n.

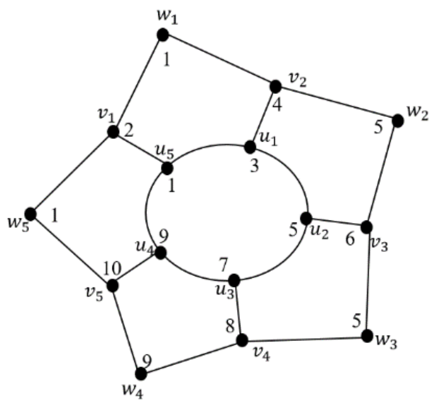

Figure 2.5. $\gamma_{\chi}^{td}(PCL_5)$=10

Theorem 2.3: Let Pc_n be a Pencil graph where $n \geq 6$. Then

$$\gamma_{\chi}^{td}(Pc_n) = \begin{cases} n+2 & if\ n \equiv 0(mod\ 2) \\ n+3 & orelse \end{cases}$$

Proof

Let V(Pc_n) =$\{u, v, u_i, v_i: 1 \leq i \leq n\}$. We take N(u)= {v,v_1,u_1}, N(v)= {u,v_n,u_n}, N(u_1)= {u, u_2, v_1}, N(v_1)={ u, v_2, u_1}, N(u_i)={ u_{i-1},u_{i+1},v_i },N(v_i)={v_{i-1},v_{i+1},u_i }, N(u_n)={v,u_{n-1},v_n}, N(v_n)= { v,v_{n-1},u_n} for all $2 \leq i \leq n-1$.

Assign color 1,2, 4i-1,4i,4j+1 and 4j+2 to the vertices {v,u_1}, {u}, {u_{4i-2},v_{4i-3},v_{4i-1}}, {v_{4i-2}},{ u_{4j-1},u_{4j+1},v_{4j}}and{u_{4j}} $\left(1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor, 1 \leq j \leq \left\lfloor \frac{n}{4} \right\rfloor - 1\right)$ respectively. We consider two cases.

Case 1: When n≡ $0(mod\ 2)$

Assign colors n-1,n, n+1 and n+2 to the vertices { u_{n-3},u_{n-1},v_{n-2}}, {u_{n-2}} , {v_{n-1}, u_n} and {v_n} respectively if n≡ 2(mod 4). The vertices {u_{n-1},v_n}and{u_n} receive colors say n+1 and n+2 respectively if n≡ 0(mod 4). Thus we obtain γ_{χ}^{td}- coloring of Pc_n. So $\gamma_{\chi}^{td}(Pc_n)$= n+2.

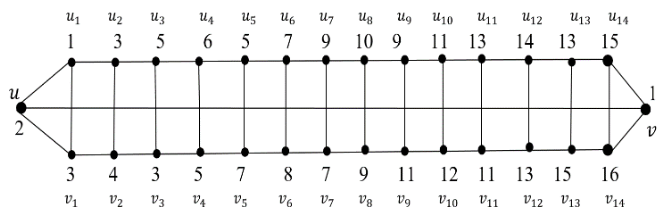

Figure 2.6. $\gamma_{\chi}^{td}(Pc_{14})$=16

Case 2: When n≡ $1(mod\ 2)$

For the vertices {u_{n-4},u_{n-2},v_{n-3}}, {u_{n-3}},{v_{n-2},v_n}, {v_{n-1}},{ u_{n-1}}and{u_n} assign colors say n-2,n-1,n,n+1,n+2 and n+3 respectively if n≡ 3(mod 4). Assign colors n, n+1,n+2 and n+3 to the vertices {u_{n-2},u_n}, {u_{n-1}}{v_{n-1}} and {v_n} respectively if n≡ 1(mod 4). Hence we attain γ_{χ}^{td}- coloring of Pc_n. So $\gamma_{\chi}^{td}(Pc_n)$= n+3.

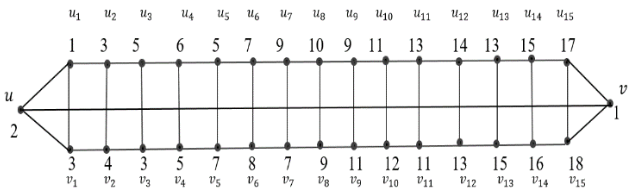

Figure 2.7. $\gamma_{\chi}^{td}(Pc_{15})$=18

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