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Bipolar Fuzzy Entropy and Correlation Co-efficient with Application to Vaccine Selection

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Abstract: The fuzzy set theory is very popular and viable approach to model the uncertainty. The uncertainty in the real cases is due to different factors like the lack of expertise, erroneous data, hesitancy, indeterminacy, existence of the counter-presence of a property. Various fuzzy set extensions offer different approaches for better modelling of the real systems in view of the underlying structural uncertainty of the real system. Bipolar fuzzy theory is also an extension of fuzzy theory that has not been investigated in much detail. In bipolar fuzzy theory, we deal with those real systems where a linguistic variable needs to be investigated for the level of satisfaction as well as counter-satisfaction. In this paper, we define certain operations on a bipolar fuzzy set and examine some of their properties. We also introduce an axiomatic framework for defining bipolar fuzzy entropy and establish a characterization theorem. We introduce three bipolar fuzzy entropy measures and a bipolar fuzzy correlation coefficient with application to a problem of multiple criteria decision-making (MCDM) that needs to be modelled with bipolar fuzzy logic.

Keywords: Fuzzy entropy; bipolar fuzzy set; bipolar fuzzy entropy; bipolar correlation; MCDM.

Background

The conventional theory due to Zadeh (1965) allows unidirectional or unipolar thinking of human judgment. Attanasov (1986) introduced a two valued representation of human judgment about some concept or aspect. Due to Attanasov's idea, intutionistic fuzzy set theory came to existence in 1986. The intuitionistic fuzzy set theory has baeen considered as more comprehensive than the classical fuzzy theory but in some restrictive sense. The two valued representation of an intuitionistic fuzzy set assumes opposite polarities of description of an aspect. Both the polarities despite being opposite receives the values in the interval [0,1] i.e., positively quantified. In real life, there are instances whose polarities are opposite and needs to be quantified in such a way that the positive polarity receives the positive value in the interval [0,1] and the negative polarity receives the negative value in the interval [-1,0]. Suppose we want to check the effect of a medicine on a patient. We know that medicine is related to specific disease. The effect of medicine on a patient will be considered as positive if it heals the patient and it will be considered as negative if the healing effect of the medicine is null but it is causing some side effect to the patient. Evaluation of certain competitive examinations also deals with two polarities. For example positive marks for correct answers and negative marks for incorrect answers. Many such circumstances exist in real life. These finding justify the co-existence of two opposite polarities with opposite sign conventions and lead to the development of bi-polar fuzzy logic. The bi-polar fuzzy logic enables the modeling of certain systems where the interaction between the two opposite polarities is desirable. Zhang (1994) introduced the concept of bi-polar fuzzy theory. Zhang (1998) considered two essential principles of fuzziness to deal with bi-polar fuzzy logic.

1. *Representation Principle*: A bi-polar fuzzy variable assumes a positive pole to understand the nature of a positive side and a negative pole to capture the essence of negative side. Further, the co-existence of both the polarities is required in the different degree of gray levels.
2. *Computational Principle*: A bi-polar fuzzy logic must support the bi-polar reasoning. Moreover, the computational interaction between the polarities are also required.

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The bipolar logic enables the modelling of certain systems where the interaction between the two opposite polarities is desirable. Bipolar fuzzy set is an extension of fuzzy set in which degree of membership is extended from closed interval [0,1] to [-1,1]. In bipolar fuzzy sets, the degree of membership 0 depicts that elements are insignificant to the corresponding property, the degrees of membership in the interval (0,1] means that the elements considerably satisfy the property and membership in the interval [-1,0) indicate that element considerably satisfy the implicit counter property. Two types of representations are used in bipolar fuzzy sets namely canonical representation and reduced representation. In the former representation, the degrees of membership are presented with a pair of positive value of membership and negative value of membership i.e., the membership degrees are divided into two portions: [0,1] for positive part and [-1,0] for negative part. Let Y be the universal set, then in canonical representation, the bipolar fuzzy set B on Y is given as:

$$B = \{(y, \mu_B^+(y), \mu_B^-(y)) : y \epsilon Y\},$$

where $\mu_B^+ : Y \rightarrow [0,1] and \mu_B^- : Y \rightarrow [-1,0]$ are positive pole and negative pole respectively. The positive pole represents the degree of satisfaction of an element y to the property corresponding to a bipolar fuzzy set B and negative pole represents the degree of satisfaction of $y$to some implicit counter-property B. The bipolar fuzzy set B in reduced representation on same universe Y is given by

$$B = \{(y, \mu_B^R(y)) : y \epsilon Y\}$$

where $\mu_B^R : Y \rightarrow [-1,1]$ denotes the membership degree.

The degree of membership $\mu_B^R(y)$ in reduced representation can be deduced using canonical representation as

$$\mu_B^R(y)) = \begin{cases} \mu_B^+(y) if \mu_B^-(y) = 0, \\ \mu_B^-(y) if \mu_B^+(y) = 0, \\ f(\mu_B^+(y), \mu_B^-(y)) otherwise. \end{cases}$$

where f is an aggregation function to merge negative as well as positive pole into a single value and such function can be of various types depending upon the domain of application.

If a bipolar fuzzy set $B = \{(y, \mu_A^+(y), \mu_A^-(y)) : y \epsilon Y\}$ is compared with intuitionistic fuzzy set $B = \{(y, \mu_B(y), \nu_B(y)) : y \epsilon Y\}$ where $\mu_B(y)$ and $\nu_B(y)$ are membership degree and non-membership degree respectively of y in Y, under assumption that $\mu_B^+(y) = \mu_B(y)$ and $\mu_B^-(y) = -\nu_B(y)$, then bipolar fuzzy set and intuitionistic fuzzy sets look similar. But they are different from one another in given ways: In bipolar fuzzy set, the positive pole, $\mu_B^+(y)$ describes the extent that the element y satisfies the property B and the negative pole, $\mu_B^-(y)$ describes the extent that the element y satisfies an implicit-counter property of B. While in intuitionistic fuzzy set, the degree of membership $\mu_B(y)$ characterizes the degree that the element y satisfies the property B and the membership degree $\nu_B(y)$ characterizes the degree that the element y satisfies the not property of B. Because counter property and not-property are not generally equivalent, so intuitionistic and bipolar fuzzy sets are non-identical fuzzy sets extensions. Along with this, the set operations union, intersection and complement also differ each other.

Related Studies

To overcome the drawbacks of unipolar system, Zhang and Zhang (2004) systematize three principles for representation and computation of bi-polarity namely Representation Principle with bi-polarity, Representation principle with bi-polarity and Fuzziness, Bipolar Computation principle. Bi-polar fuzzy sets are different from intuitionistic fuzzy sets. Lee and LEE (2001) compared bipolar fuzzy valued sets with intuitionistic fuzzy sets. Chen and et al. (2014) extended bipolar fuzzy sets to m-Polar fuzzy sets. Patrascu (2015) introduced similarity measure and distance measure in terms of penta-valued representation of bipolar fuzzy sets. Zarasiz and Riaz (2022) put forward bipolar metric space and studied some topological as well as functional properties of bipolar metric space. Riaz and Tehrim (2019a) presented internal cubic bipolar fuzzy sets (ICBF-sets), external cubic bipolar fuzzy sets (ECBF-sets), aggregation operators to aggregate ICBF and ECBF data along with application in MCDM problem. Also, Riaz and Tehrim (2019b) introduced interval valued bipolar fuzzy sets and Cubic bipolar fuzzy set along with its application in multi criteria group decision making. Some operations like concentration, dilation, support, core, binary relations and correlation coefficient are defined by Riaz et al.

(2021). Also Riaz and Tehrim (2020) presented some weighted geometric aggregation operators for CBFSs under dual order. Bipolar picture fuzzy sets and some aggregation operators and distance measures on bipolar picture fuzzy sets with application in MCDM were also discussed by Riaz and et al. (2021). Mahmood et al. (2016) discussed multi-criteria decision-making problem of bipolar-valued fuzzy. Bipolar fuzzy graphs and its applications are introduced by Akram (2011), (2013). Akram and Karunambigai (2011) defined metric on bipolar fuzzy graphs. Application of bipolar fuzzy graphs in decision making problems is done by Akram and Waseem (2018). Akram and et al. (2020) discussed the comparison of bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE-I method and used them in medical diagnosis. Singh(2022) measures the information weights in bipolar fuzzy concept using Shannon's entropy and also differentiate bipolar fuzzy set from intutionistic fuzzy set.

Motivation and contribution: The scenario regarding the effect of medicine, evaluation of competitive exams, cooperation in a joint project, etc., need to be addressed by bipolar fuzzy logic. This non-standard version or extension of fuzzy logic has very limited number of studies to date. No significant information theoretic measures have been proposed in the bipolar fuzzy settings. In a nutshell, concerning bipolar fuzzy logic, to the best of our knowledge, very limited number of theoretical and practical studies is available in the literature. These possibilities of extension of bipolar fuzzy logic motivated us to study the fundamental operations regarding the bipolar fuzzy sets, some properties and the development of information theoretic measures concerning bipolar fuzzy sets.

The main contribution of this paper is as follows:

1. We define some fundamental operations on bipolar fuzzy sets.
2. We also define linguistic hedges on a bipolar fuzzy set.
3. We proposed entropy of a bipolar fuzzy set along with some characterization results.
4. We apply our proposed measure in multi attribute decision making (MADM) problem with a bipolar fuzzy knowledge-base.

The context of this paper is assembled as follows:

Section 2 consists of relevant basic definitions regarding the bipolar fuzzy sets. Some operations on bipolar fuzzy sets along with some example have been presented in section 3. An axiomatic definition for bipolar fuzzy entropy, Characterization theorem and some bipolar fuzzy entropy measure are presented in the section 4. Section 5 defines correlation coefficient between two bipolar fuzzy sets. In the succeeding section (i.e., in section 6) a TOPSIS method in bipolar fuzzy environment is introduced. In section 7, the proposed TOPSIS method is applied to an example when weights of criteria are partially known or completely unknown. At the end, conclusion is presented in the section 8.

Preliminaries

In this section, we present some basic concepts related to bipolar fuzzy theory.

Definition: **(Lee and LEE 2001)**

Let X=$\{x_1,x_2, x_3,\ldots, x_n\}$ be universe of discourse and consider a mapping A_μ:X→[0,1]×[-1,0] by

A_μ(x)=$(\mu_A^+(x), \mu_A^-(x))$.

Then the set A=$\{(\mu_A^+(x), \mu_A^-(x))$: $x\epsilon X$ $\}$ is known as Bipolar fuzzy set on X.

Definition: **(Lee and LEE 2001)**

The complement of a bipolar fuzzy set A in X, where X is universe of discourse, is defined by

A^C=$\{(1-\mu_A^+(x), -1-\mu_A^-(x))$:$x\epsilon X$ $\}$

Definition: **(Lee and LEE 2001)**

Let A = $\{(\mu_A^+(x), \mu_A^-(x))$: $x\epsilon X$ $\}$ and B=$\{(\mu_B^+(x), \mu_B^-(x))$: $x\epsilon X$ $\}$ be two bipolar fuzzy sets on universe of discourse X, then union of these two bipolar fuzzy sets is defined as

$$A \cup B = \{(\max\{\mu_A^+(x), \mu_B^+(x)\}, \min\{\mu_A^-(x), \mu_B^-(x)\}) : x \epsilon X\}.$$

Definition: **(Lee and LEE 2001)**

Let $A = \{(\mu_A^+(x), \mu_A^-(x)) : x\epsilon X\}$ and $B = \{(\mu_B^+(x), \mu_B^-(x)) : x\epsilon X\}$ be two bipolar fuzzy sets on universe of discourse X, then intersection of these two bipolar fuzzy sets is defined as

$$A \cap B = \{(\min\{\mu_A^+(x), \mu_B^+(x)\}, \max\{\mu_A^-(x), \mu_B^-(x)\}) : x\epsilon X\}.$$

In the conventional fuzzy theory, crispness of a set means the membership degree of elements of the universal set is either 0 or 1. While dealing with the bipolar fuzzy logic, the presence of an element of the universal set in the bipolar fuzzy set can be thought of in the positive sense or in the negative sense. Therefore, we have an extended notion of crispness that considers the complete presence of element of universal set in positive sense or in negative sense. Mathematically, in bipolar fuzzy representation, these two situations can be represented as $(1,0)$ and $(0,-1)$ respectively. Clearly, $(1,0)$ and $(0,-1)$ are complement of each other in bipolar fuzzy logic. Further, in conventional fuzzy theory, a set A is most fuzzy if each element of the universal set has membership value equal to 0.5. In the bipolar fuzzy logic, this situation is equivalent to the fact that the sum of positive polarity and the negative polarity is equals to zero for each element of the universal set. Also, if A and B are two bipolar fuzzy sets on a universe X, then A will be less fuzzy than B in two ways:

(i) $1 \geq \mu_A^+(x) + \mu_A^-(x) \geq \mu_B^+(x) + \mu_B^-(x) \geq 0$

(ii) $-1 \leq \mu_A^+(x) + \mu_A^-(x) \leq \mu_B^+(x) + \mu_B^-(x) \leq 0$ for all $x\epsilon X$.

Riaz and et al. (2021) defined R-concentration, P-concentration, R-dilation, P-dilation, support, core for cubic bipolar fuzzy sets. Cubic bipolar fuzzy set is an extension of bipolar fuzzy set and using the operations defined on cubic bipolar fuzzy sets, we can define concentration, dilation, linguistic hedges for ordinary bipolar fuzzy sets as under:

Let A be a bipolar fuzzy set on universe of discourse X. Then

1. ***Concentration***

The Concentration of a bipolar fuzzy set A in the universe of discourse X, denoted by $\mathrm{CON}(A)$ and is defined as

$$\mathrm{CON}(A) = \{((\mu_A^+(x_i))^2, -1 + \left(1 + \mu_A^-(x_i)\right)^2) : x_i\epsilon X\}$$

i.e., A^2 is considered as concentration of A.

2. ***Dilation***

The Concentration of a bipolar fuzzy set A in the universe of discourse X, denoted by $\mathrm{DIL}(A)$ and is defined as

$$\mathrm{DIL}(A) = \{((\mu_A^+(x_i))^{\frac{1}{2}}, -1 + \left(1 + \mu_A^-(x_i)\right)^{\frac{1}{2}}) : x_i\epsilon X\}$$

i.e., $\mathrm{DIL}(A) = A^{\frac{1}{2}}$.

If B is a bipolar fuzzy set the $\mathrm{CON}(B)$ may be considered as very(B) and $\mathrm{DIL}(A)$ may be treated as more or less(B).

3. ***Linguistic Modifier***

$$A^n = \{((\mu_A^+(x_i))^{\mathrm{n}}, -1 + \left(1 + \mu_A^-(x_i)\right)^{\mathrm{n}}) : x_i\epsilon X\} \text{ where } n > 0.$$

4. $\square A = \left\{\left(\mu_A^+(x_i), -1 + \mu_A^+(x_i)\right) : x_i\epsilon X\right\}$.

Example: Let $A = \{(0.2, -0.8), (0.4, -0.5), (0.6, -0.2), (0.45, -0.5), (0.26, -0.58)\}$ be a bipolar fuzzy set on universe $X = \{a, b, c, d, e\}$.

Then, $\square A = \{(0.2, -0.8), (0.4, -0.6), (0.6, -0.4), (0.45, -0.55), (0.26, -0.74)\}$.

5. $\diamond A = \{(1 + \mu_A^-(x_i), \mu_A^-(x_i)) : x_i \epsilon X\}$.

Example: Let $A = \{(0.2, -0.8), (0.4, -0.5), (0.6, -0.2), (0.45, -0.5), (0.26, -0.58\ )\}$ be a bipolar fuzzy set on universe $X = \{a, b, c, d, e\}$.

Then $\diamond A = \{(0.2, -0.8), (0.5, -0.5), (0.8, -0.2), (0.5, -0.5), (0.42, -0.58)\}$.

Example 1: Suppose $X = \{a, b, c, d, e\}$ be the universe of discourse. A bipolar fuzzy set "BIG" of X may be defined as

BIG $= \{(0.2, -0.6), (0.08, -0.9), (0.54, -0.54), (0.35, -0.20), (0.74, -0.92), (0.85, -0.73)\}$.

Thus, Very BIG

$$= \{(0.04, -0.84), (0.006, -0.99), (0.29, -0.78), (0.12, -0.36), (0.54, -0.99), (0.72, -0.92)\}.$$

Not Very BIG

$$= \{(0.8, -0.4), (0.92, -0.1), (0.46, -0.46), (0.65, -0.8), (0.26, -0.08), (0.15, -0.27)\}.$$

Very Very BIG

$= \{(0.001, -0.97), (0, -0.99), (0.08, -095), (0.01, -0.59), (0.29, -0.99), (0.52, -0.99)\}$.

Thus, Very BIG

$= \{(0.44, -0.36), (0.28, -0.68), (0.73, -0.32), (0.59, -0.10), (0.86, -0.71), (0.92, -0.48)\}$.

Example of operation A^n:

Let $A = \{(0,0), (0.4, -0.4), (0.6, -0.6), (0.75, -0.75), (0.8, -0.8)$,(0.75,-0.75), (0.6,-0.6), (0.4,-0.4), (0,0)} be a bipolar fuzzy set on universe of discourse $X = \{1,2, \dots ,9\}$.

Then

$A^{\frac{1}{2}} = \{(0,0), (0.63246, -0.2254), (0.7745, -0.3675), (0.86603, -0.5)$,$(0.89443, -0.5528)$,

$(0.86603, -0.5), (0.7745, -0.3675), (0.63246, -0.2254), (0,0)\}$;

$A^2 = \{(0,0), (0.16, -0.64), (0.36, -0.84), (0.5625, -0.9475), (0.64, -0.96)$,

$(0.5625, -0.9475), (0.36, -0.84), (0.16, -0.64), (0,0)\}$;

$A^3 = \{(0,0), (0.064, -0.784), (0.216, -0.936), (0.42188, -0.9844), (0.512, -0.992)$,

(0.42188,−9844), (0.216,−0.936), (0.064,−0.784),(0,0)};

$A^4 = \{(0,0), (0.256, -0.8704), (0.1296, -0.9744), (0.31641, -0.9961)$,

$(0.4096, -0.9984), (0.31641, -0.9961), (0.1296, -0.9784)$,

$(0.256, -0.8704), (0,0)\}$.

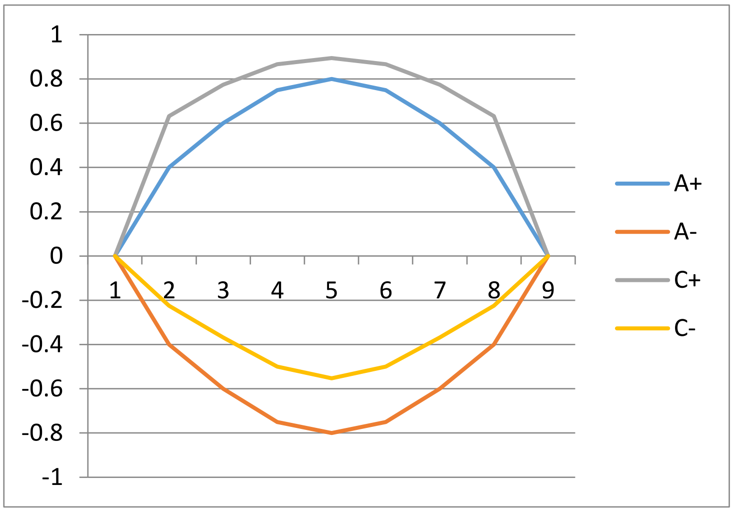

Figure 1. Concentration of bipolar fuzzy set A.

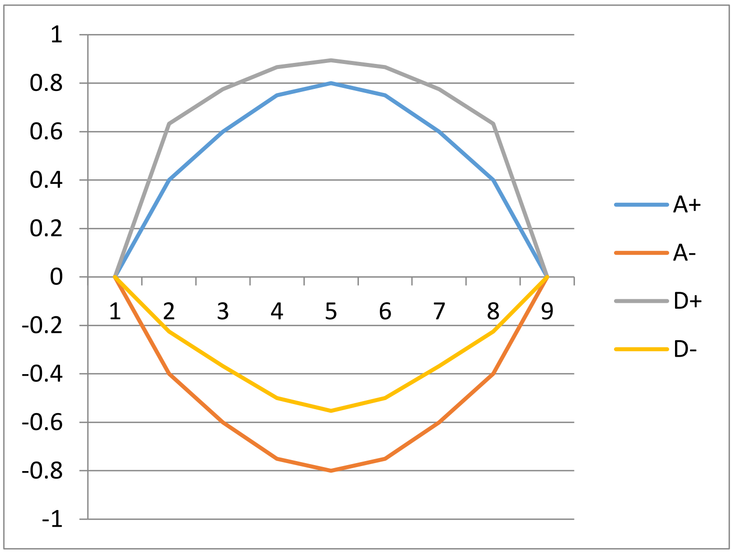

Figure 2. Dilation of bipolar fuzzy set A.

In figure 1, A+ and A- represents the positive pole and negative pole respectively of bipolar fuzzy set A, C+ and C- represents the positive pole and negative pole of Concentration of A respectively.

In figure 2, A+ and A- represents the positive pole and negative pole respectively of bipolar fuzzy set A, D+ and D- represents the positive pole and negative pole of Dilation of A respectively.

Theorem 1: Let A be any bipolar fuzzy set in a universe X, then

$(i) \square A^n = (\square A)^n$;

$$(ii) \diamond A^n = (\diamond A)^n.$$

Proof: From the above operations on bipolar fuzzy sets, the proof is just an easy calculation.

Theorem 2: For any two bipolar fuzzy set A on X, we have

$(i) \square \mathrm{CON}(A) = \mathrm{CON}(\square A)$

$(ii) \diamond \mathrm{CON}(A) = \mathrm{CON}(\diamond A)$

$(iii) \square \mathrm{DIL}(A) = \mathrm{DIL}(\square A)$

$(iv) \diamond \mathrm{DIL}(A) = \mathrm{DIL}(\diamond A)$

Proof: $(i)\square\mathrm{CON}(A) = \square\{((\mu_A^+(x_i))^2, -1+\left(1+\mu_A^-(x_i)\right)^2): x_i\epsilon X\}$

$= \{((\mu_A^+(x_i))^2, -1+\left(\mu_A^+(x_i)\right)^2): x_i\epsilon X\}$.

and $\mathrm{CON}(\square A) = \mathrm{CON}\{(\mu_A^+(x_i), -1+\mu_A^+(x_i)): x_i\epsilon X\}$

$= \{((\mu_A^+(x_i))^2, -1+\left(\mu_A^+(x_i)\right)^2): x_i\epsilon X\}$.

This proves (i).

$(ii)\ \diamond\mathrm{CON}(A) = \diamond\ \{((\mu_A^+(x_i))^2, -1+\left(1+\mu_A^-(x_i)\right)^2): x_i\epsilon X\}$

$= \{((1+\mu_A^-(x_i))^2, -1+\left(1+\mu_A^-(x_i)\right)^2): x_i\epsilon X\}$.

and $\mathrm{CON}(\diamond A) = \mathrm{CON}\{(1+\mu_A^-(x_i), \mu_A^-(x_i)): x_i\epsilon X\}$

$= \{((1+\mu_A^-(x_i))^2, -1+\left(1+\mu_A^-(x_i)\right)^2): x_i\epsilon X\}$.

Thus, $\diamond\mathrm{CON}(A) = \mathrm{CON}(\diamond A)$.

$(iii)\square\mathrm{DIL}(A) = \square\{((\mu_A^+(x_i))^{\frac{1}{2}}, -1+\left(1+\mu_A^-(x_i)\right)^{\frac{1}{2}}): x_i\epsilon X\}$

$= \{((\mu_A^+(x_i))^{\frac{1}{2}}, -1+\left(\mu_A^+(x_i)\right)^{\frac{1}{2}}): x_i\epsilon X\}$.

and $\mathrm{DIL}(\square A) = \mathrm{DIL}\{(\mu_A^+(x_i), -1+\mu_A^+(x_i)): x_i\epsilon X\}$

$= \{((\mu_A^+(x_i))^{\frac{1}{2}}, -1+\left(\mu_A^+(x_i)\right)^{\frac{1}{2}}): x_i\epsilon X\}$.

This proves (iii).

$(iv)\ \diamond\mathrm{DIL}(A) = \diamond\ \{((\mu_A^+(x_i))^{\frac{1}{2}}, -1+\left(1+\mu_A^-(x_i)\right)^{\frac{1}{2}}): x_i\epsilon X\}$

$= \{((1+\mu_A^-(x_i))^{\frac{1}{2}}, -1+\left(1+\mu_A^-(x_i)\right)^{\frac{1}{2}}): x_i\epsilon X\}$.

and $\mathrm{DIL}(\diamond A) = \mathrm{DIL}\{(1+\mu_A^+(x_i), \mu_A^-(x_i)): x_i\epsilon X\}$

$= \{((1+\mu_A^-(x_i))^{\frac{1}{2}}, -1+\left(1+\mu_A^-(x_i)\right)^{\frac{1}{2}}): x_i\epsilon X\}$.

Thus, $\diamond\mathrm{DIL}(A) = \mathrm{DIL}(\diamond A)$.

In the next section, we introduce the notion of bipolar fuzzy entropy.

4. Bipolar fuzzy entropy

4.1Axiomatic frame work and characterization theorem

Luca and Termini (1972) introduced the concept of non-probabilistic entropy in fuzzy set theory. We construct an axiomatic frame work for bipolar fuzzy entropy in the sense of classical fuzzy entropy by consideration of crispness, most fuzziness, complement and relative fuzziness of bipolar fuzzy environment (as discussed in section 2).

Definition

A real valued function E: $\mathrm{BFS}(X) \rightarrow [0,1]$ is called as knowledge measure on $\mathrm{BFS}(X)$ if it satisfies the following properties:

BFE1:$\mathrm{E}(A) = 0$ if A is a crisp set i.e., if $\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (1,0)$ or$\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (0,-1)$ for all $x_i\epsilon X$.

BFE2: $\mathrm{E}(A) = 1$ if A is most fuzzy set i.e., if $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$ for all $x_i\epsilon X$.

BFE3: $\mathrm{E}(A) = \mathrm{E}(A^C)$.

BFE4: $\mathrm{E}(A) \leq \mathrm{E}(B)$ if either -1 $\leq \mu_A^+(x_i) + \mu_A^-(x_i) \leq \mu_B^+(x_i) + \mu_B^-(x_i) \leq 0$

or $1 \geq \mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i) \geq 0$ for all $x_i \epsilon X$.

Theorem 3 (Characterization Theorem)

Let $h: [0,1] \times [-1,0] \to [0,1]$ be a mapping. Then the function $E: BF(X) \to [0,1]$

defined by

$E(A) = \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big)$,satisfies:

(1) $E(A) = 0$ if A is a crisp set i.e., if $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (1,0)$ or $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (0,-1)$ for all $x_i \epsilon X$.

(2) $E(A) = 1$ iff $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$ for all $x_i \epsilon X$.

(3) $E(A) = E(A^C)$.

(4) $E(A) \leq E(B)$ if either $-1 \leq \mu_A^+(x_i) + \mu_A^-(x_i) \leq \mu_B^+(x_i) + \mu_B^-(x_i) \leq 0$

or $1 \geq \mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i) \geq 0$ for all $x_i \epsilon X$.

if and only if

$(i) h(x,y) = 0$ iff $(x,y) = (1,0)$ or $(x,y) = (0,-1)$;

$(ii) h(x,y) = 1$ iff $x + y = 0$;

$(iii) h(x,y) = h(1-x, -1-y)$;

$(iv) h(x,y) \leq h(z,w)$ if either $-1 \leq x + y \leq z + w \leq 0$ or $1 \geq x + y \geq z + w \geq 0$ for all $x \epsilon X$.

***Proof* :** Suppose that E satisfies(1)-(4). We prove (i)-(iv).

(i) Suppose $h(x,y) = 0; x\epsilon[0,1],\ y\epsilon[-1,0]$. Taking $x = \mu_A^+(x_i)$ and $y = \mu_A^-(x_i)$ for all $i = 1,2,3,\dots,n$. Then for the resulting bipolar fuzzy set A, we have

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0$$

which is possible if and only if $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (1,0)$ or

$\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (0,-1)$,

i.e., $h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0$ if and only if $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (1,0)$ or

$\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (0,-1)$.

or $h(x,y) = 0$ iff $(x,y) = (1,0)$ or $(x,y) = (0,-1)$.

(ii) Now $E(A) = \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 1$iff $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$ for all $i = 1,2,\dots,n$.

$\Rightarrow h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 1$ iff $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$ for all $i = 1,2,\dots,n$.

or $h(x,y) = 1$ iff $x + y = 0; x\epsilon[0,1],\ y\epsilon[-1,0]$.

(iii) Let us suppose there exists $x\epsilon[0,1] and y\epsilon[-1,0]$ such that

$h(x,y) \neq h(1-x, -1-y)$.

Without any loss of generality, let us assume that

$h(x,y) < h(1-x, -1-y)$.

Consider the bipolar fuzzy set, A given by $\mu_A^+(x_i) = x$and $\mu_A^-(x_i) = y$, for all i.

Then $h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) < h\big(1 - \mu_A^+(x_i), -1 - \mu_A^-(x_i)\big)$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) < \frac{1}{n}\sum_{i=1}^{n} h\big(1-\mu_A^+(x_i), -1-\mu_A^-(x_i)\big)$

$\Rightarrow E(A) < E(A^C)$, which contradicts (3).

Thus our supposition is wrong and (iii) follows.

(iv) Take $\mu_A^+(x_i) = x, \mu_A^-(x_i) = y, \mu_B^+(x_i) = z, \mu_B^-(x_i) = w$.

Suppose there exists i such that $1 \geq \mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i) \geq 0$,

but $h(\mu_A^+(x_i), \mu_A^-(x_i)) > h(\mu_B^+(x_i), \mu_B^-(x_i))$.

Considering for all other $i's$, $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0 = (\mu_B^+(x_i), \mu_B^-(x_i))$, then

$h(\mu_A^+(x_i), \mu_A^-(x_i)) > h(\mu_B^+(x_i), \mu_B^-(x_i))$

$\Rightarrow E(A) > E(B)$, which is a contradiction to (4).

Similarly, if we take $-1 \leq \mu_A^+(x_i) + \mu_A^-(x_i) \leq \mu_B^+(x_i) + \mu_B^-(x_i) \leq 0$, then we will again arrive at contradiction.

Thus, $h(x, y) \leq h(z, w)$ if either $-1 \leq x + y \leq z + w \leq 0$ or $1 \geq x + y \geq z + w \geq 0$.

Conversely, suppose that h satisfies (i)-(iv), we show that (1)-(4) holds.

$E(A) = \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big)$ satisfies (1)-(4).

(i) Suppose that $\big(\mu_A^+(x), \mu_A^-(x)\big) = (1,0)$ or $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (0,-1)$, for all $i = 1,2,\dots,n$;

Where A is a bipolar fuzzy set. Then

$$h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0$$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0$

$\Rightarrow E(A) = 0$

Conversely, suppose that $E(A) = 0$, then

$$\frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0$$

$\Rightarrow h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0$

$\Rightarrow \big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (1,0)$ or $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (0,-1)$.

(ii) Suppose $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2,\dots,n$. Then

$h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 1$, for all $i = 1,2,\dots,n$.

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 1$

$\Rightarrow E(A) = 1$.

Conversely, suppose that $E(A) = 1$, then

$$\frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 0$$

$\Rightarrow h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = 1$, for all $i = 1,2,\dots,n$.

$\Rightarrow \mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2,\dots,n$.

(iii) Consider $E(A^C) = \frac{1}{n}\sum_{i=1}^{n} h\big(1-\mu_A^+(x_i), -1-\mu_A^-(x_i)\big)$

$= \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = E(A).$

Thus, $E(A) = E(A^C)$.

(iv)Suppose $1 \geq \mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i) \geq 0$, for all $i = 1,2,\dots,n$.

Or or $1 \geq x + y \geq z + w \geq 0$, where $\mu_A^+(x_i) = x, \mu_A^-(x_i) = y, \mu_B^+(x_i) = z, \mu_B^-(x_i) = w$. Then

$$h(x,y) \leq h(z,w)$$

or $h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) \leq h\big(\mu_B^+(x_i), \mu_B^-(x_i)\big)$, for all $i = 1,2,\dots,n$.

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) \leq \frac{1}{n}\sum_{i=1}^{n} h\big(\mu_A^+(x_i), \mu_A^-(x_i)\big)$

$\Rightarrow \qquad E(A) \leq E(B).$

Similarly for $-1 \leq \mu_A^+(x_i) + \mu_A^-(x_i) \leq \mu_B^+(x_i) + \mu_B^-(x_i) \leq 0$, we have $E(A) \leq E(B)$.

In the next section, we propose some entropy measures in bipolar fuzzy settings.

4.2 *Some bipolar fuzzy entropy measures*

Let $A \epsilon BF(X)$, then we propose the following bipolar fuzzy entropy measures.

$$E_1(A) = 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right).$$

$$E_2(A) = \frac{1}{n}\sum_{i=1}^{n} \cos\left(\big(\mu_A^+(x_i) + \mu_A^-(x_i)\big)\frac{\pi}{2}\right).$$

$$E_3(A) = \frac{1}{n}\sum_{i=1}^{n} [(\cos\left(\big(\mu_A^+(x_i) + \mu_A^-(x_i)\big)\frac{\pi}{4}\right) - \frac{1}{\sqrt{2}})\frac{\sqrt{2}}{\sqrt{2}-1}].$$

Theorem 4

$E_1(A) = 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right)$ ia a valid bipolar fuzzy entropy measure.

***Proof*:** We establish the following axioms **BFE1-BFE4.**

BFE1: First suppose that $\big(\mu_A^+(x), \mu_A^-(x)\big) = (1,0)$ or $\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (0,-1)$, for all $i = 1,2,\dots,n$. Then $E_1(A) = 0$.

Conversely, suppose that $E_1(A) = 0$, then

$$\frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right) = 1$$

$\Rightarrow \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow |\mu_A^+(x_i) + \mu_A^-(x_i)| = 1$

$\Rightarrow \big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (1,0)or\big(\mu_A^+(x_i), \mu_A^-(x_i)\big) = (0,-1)$ for all $x_i \epsilon X$.

This proves **BFE1.**

BFE2: First suppose that $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2,\dots,n$. Then $E_1(A) = 1$.

Conversely, suppose that $E_1(A) = 1$, then

$$-\frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right) = 0$$

$$\Rightarrow \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right) = 0,\ i = 1,2,\dots,n.$$

$\Rightarrow |\mu_A^+(x_i) + \mu_A^-(x_i)| \frac{\pi}{2} = m\pi$, m is an integer

$\Rightarrow |\mu_A^+(x_i) + \mu_A^-(x_i)| = 2m$

$\Rightarrow m \geq 0$ but $m \geq 1$is not possible.

$\Rightarrow |\mu_A^+(x_i) + \mu_A^-(x_i)| = 0$

$\Rightarrow \mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2, \dots, n$.

BFE3: It follows easily.

BFE4: Let $1 \geq \mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i) \geq 0$, for all i=1,2,…,n. Then we have

$$|\mu_A^+(x_i) + \mu_A^-(x_i)| \geq |\mu_B^+(x_i) + \mu_B^-(x_i)|$$

$\Rightarrow \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)| \frac{\pi}{2}\right) \geq \sin\left(|\mu_B^+(x_i) + \mu_B^-(x_i)| \frac{\pi}{2}\right)$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)| \frac{\pi}{2}\right) \geq \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu_A^+(x_i) + \mu_A^-(x_i)| \frac{\pi}{2}\right)$

$\Rightarrow E_1(A) \leq E_1(B)$.

Similarly, if $-1 \leq \mu_A^+(x_i) + \mu_A^-(x_i) \leq \mu_B^+(x_i) + \mu_B^-(x_i) \leq 0$, we have

$$E_1(A) \leq E_1(B).$$

Thus $E_1(A)$ is valid bipolar fuzzy entropy.

Theorem 5

$E_2(A) = \frac{1}{n}\sum_{i=1}^{n} \cos\left((\mu_A^+(x_i) + \mu_A^-(x_i)) \frac{\pi}{2}\right)$ is a valid bipolar fuzzy entropy measure.

Proof: We establish the following axioms **BFE1-BFE4.**

BFE1: First suppose that $\left(\mu_A^+(x), \mu_A^-(x)\right) = (1,0)$ or $\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (0, -1)$, for all $i = 1,2, \dots, n$. Then $E_2(A) = 0$.

Conversely, suppose that $E_1(A) = 0$, then $\frac{1}{n}\sum_{i=1}^{n} \cos\left((\mu_A^+(x_i) + \mu_A^-(x_i)) \frac{\pi}{2}\right) = 0$

$\Rightarrow \cos\left((\mu_A^+(x_i) + \mu_A^-(x_i)) \frac{\pi}{2}\right) = 0$, for all $i = 1,2, \dots, n$.

$\Rightarrow$ $(\mu_A^+(x_i) + \mu_A^-(x_i))\frac{\pi}{2} = (2m+1)\frac{\pi}{2}$, for all $i = 1,2, \dots, n$; and m is an integer.

$\Rightarrow$ But m$\geq$ 1 and $m < -1$ is not possible. Therefore $m = 0$ or $m = -1$.

$\Rightarrow \mu_A^+(x_i) + \mu_A^-(x_i) = 1$ or -1, for all $i = 1,2, \dots, n$.

$\Rightarrow \left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (1,0)or\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (0, -1)$ for all $i = 1,2, \dots, n$.

This proves **BFE1.**

BFE2: First suppose that $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2, \dots, n$. Then $E_2(A) = 1$.

Conversely, suppose that $E_2(A) = 1$, then

$$\frac{1}{n}\sum_{i=1}^{n} \cos\left((\mu_A^+(x_i) + \mu_A^-(x_i)) \frac{\pi}{2}\right) = 1$$

$\Rightarrow \cos\left(\left(\mu_A^+(x_i) + \mu_A^-(x_i)\right) \frac{\pi}{2}\right) = 1$, $i = 1,2, \dots, n$.

$\Rightarrow (\mu_A^+(x_i) + \mu_A^-(x_i)) \frac{\pi}{2} = 0$, for all $i = 1,2, \dots, n$.

$\Rightarrow \mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2, \dots, n$.

BFE3: It follows easily.

BFE4: Let $1 \geq \mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i) \geq 0$, for all i=1,2,…,n. Then we have

$$\mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i)$$

$$\Rightarrow \qquad \cos\left((\mu_A^+(x_i) + \mu_A^-(x_i))\frac{\pi}{2}\right) \leq \cos\left((\mu_B^+(x_i) + \mu_B^-(x_i))\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\cos\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right) \leq \frac{1}{n}\sum_{i=1}^{n}\cos\left(|\mu_A^+(x_i) + \mu_A^-(x_i)|\frac{\pi}{2}\right)$$

$\Rightarrow E_2(A) \leq E_2(B)$.

Similarly, if $-1 \leq \mu_A^+(x_i) + \mu_A^-(x_i) \leq \mu_B^+(x_i) + \mu_B^-(x_i) \leq 0$, we have

$$E_2(A) \leq E_2(B).$$

Thus $E_2(A)$ is valid bipolar fuzzy entropy.

Theorem 6

$E_3(A) = \frac{1}{n}\sum_{i=1}^{n}[(\cos\left((\mu_A^+(x_i) + \mu_A^-(x_i))\frac{\pi}{4}\right) - \frac{1}{\sqrt{2}})\frac{\sqrt{2}}{\sqrt{2}-1}]$ is a valid bipolar fuzzy entropy.

Proof:Similar proof as above with some calculations.

Theorem 7

A mapping $E_b: BF(X) \to [0,1]$ satisfies

$(i)E_b(A) = 0$ iff$\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (1,0)or\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (0,-1)$ for all $i = 1,2,\dots,n$.

$(ii)E_b(A) = 1$ iff $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2,\dots,n$.

$(iii)E_b(A) = E_b(A^C)$.

$(iv)E_b(A) \leq E_b(B)$ if either $1 \geq \mu_A^+(x_i) + \mu_A^-(x_i) \geq \mu_B^+(x_i) + \mu_B^-(x_i) \geq 0$

or $-1 \leq \mu_A^+(x_i) + \mu_A^-(x_i) \leq \mu_B^+(x_i) + \mu_B^-(x_i) \leq 0$

if and only if for every automorphism $\varphi: [0,1] \to [0,1]$, the mapping $\varphi(E_b)$ also satisfies (i)-(iv).

Proof **:**Suppose $\varphi(E_b)$ satisfies (i)-(iv).

Because an automorphism is a strictly increasing function such that $\varphi(0) = 0$ and $\varphi(1) = 1$,

when $\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (1,0)or\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (0,-1)$ for all $i = 1,2,\dots,n$.

Then $E_b(A) = 0$

$\Rightarrow \varphi(E_b) = 0$,

And when $\varphi(E_b) = 0$, we have $E_b(A) = 0$

$\Rightarrow \left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (1,0)or\left(\mu_A^+(x_i), \mu_A^-(x_i)\right) = (0,-1)$ for all $i = 1,2,\dots,n$.

This proves that φ satisfies (i).

Now, when $\mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2,\dots,n$.

Then $E_b(A) = 1$

$\Rightarrow \varphi(E_b) = 1$

and when $\varphi(E_b) = 1$, we have $E_b(A) = 1$

$\Rightarrow \mu_A^+(x_i) + \mu_A^-(x_i) = 0$, for all $i = 1,2,\dots,n$.

This proves that φ satisfies (ii).

Since, $E_b(A) = E_b(A^C)$, it follows that φ satisfies (iii) and from the strict monotonicity of φ, it is easy to see that φ satisfies (iv).

For converse, it is enough to consider the automorphism $\varphi(x) = x$ and since, φ satisfies (i)-(iv), so for $\varphi(x) = x$, clearly E_b satisfies (i)-(iv).

Theorem 8 Let A_1 and B_1 be two bipolar fuzzy sets defined on $X = \{x_1, x_2, \ldots, x_n\}$ where

$A_1 = \{(\mu^+_{A_1}(x_i), \mu^-_{A_1}(x_i)) : x_i \epsilon X\}$ and $B_1 = \{(\mu^+_{B_1}(x_i), \mu^-_{B_1}(x_i)) : x_i \epsilon X\}$ such that they satisfy for any $x_i \epsilon X$ either $A_1 \subseteq B_1$ or $B_1 \subseteq A_1$, then

$E_1(A_1 \cup B_1) + E_1(A_1 \cap B_1) = E_1(A_1) + E_1(B_1)$,

where $E_1(A_1) = 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu^+_{A_1}(x_i) + \mu^-_{A_1}(x_i)|\frac{\pi}{2}\right)$.

***Proof*:** Partition $X = \{x_1, x_2, \ldots, x_n\}$ into two parts X_1 and X_2 such that $X_1 = \{x_i \epsilon X : A_1 \subseteq B_1\}$ and $X_2 = \{x_i \epsilon X : B_1 \subseteq A_1\}$.

Therefore

$$E_1(A_1 \cup B_1) = 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu^+_{A_1\cup B_1}(x_i) + \mu^-_{A_1\cup B_1}(x_i)|\frac{\pi}{2}\right).$$

$$= 1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{A_1\cup B_1}(x_i) + \mu^-_{A_1\cup B_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{A_1\cup B_1}(x_i) + \mu^-_{A_1\cup B_1}(x_i)|\frac{\pi}{2}\right)\right]$$

$$= 1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{B_1}(x_i) + \mu^-_{B_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{A_1}(x_i) + \mu^-_{A_1}(x_i)|\frac{\pi}{2}\right)\right]$$

And

$$E_1(A_1 \cap B_1) = 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu^+_{A_1\cap B_1}(x_i) + \mu^-_{A_1\cap B_1}(x_i)|\frac{\pi}{2}\right).$$

$$= 1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{A_1\cap B_1}(x_i) + \mu^-_{A_1\cap B_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{A_1\cap B_1}(x_i) + \mu^-_{A_1\cap B_1}(x_i)|\frac{\pi}{2}\right)\right]$$

$$= 1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{A_1}(x_i) + \mu^-_{A_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{B_1}(x_i) + \mu^-_{B_1}(x_i)|\frac{\pi}{2}\right)\right]$$

Consider

$E_1(A_1 \cup B_1) + E_1(A_1 \cap B_1)$

$$= 1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{B_1}(x_i) + \mu^-_{B_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{A_1}(x_i) + \mu^-_{A_1}(x_i)|\frac{\pi}{2}\right)\right] + 1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{A_1}(x_i) + \mu^-_{A_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{B_1}(x_i) + \mu^-_{B_1}(x_i)|\frac{\pi}{2}\right)\right.$$

$$= 1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{A_1}(x_i) + \mu^-_{A_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{A_1}(x_i) + (x_i)|\frac{\pi}{2}\right)\right] +$$

$$1 - \frac{1}{n}\left[\sum_{X_1} \sin\left(|\mu^+_{B_1}(x_i) + \mu^-_{B_1}(x_i)|\frac{\pi}{2}\right) + \sum_{X_2} \sin\left(|\mu^+_{B_1}(x_i) + \mu^-_{B_1}(x_i)|\frac{\pi}{2}\right)\right]$$

$$= 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu^+_{A_1}(x_i) + \mu^-_{A_1}(x_i)|\frac{\pi}{2}\right) + 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left(|\mu^+_{B_1}(x_i) + \mu^-_{B_1}(x_i)|\frac{\pi}{2}\right).$$

$= E_1(A_1) + E_1(B_1)$.

Hence proved.

Correlation coefficient of bipolar fuzzy sets:

Let $E$and F be two bipolar fuzzy sets of universe X. The correlation coefficient denoted by $\theta(\mathrm{E},\mathrm{F})$ is defined

$$\theta(\mathrm{E},\mathrm{F})=\frac{\curlywedge(\mathrm{E},\mathrm{F})}{\sqrt{\alpha(\mathrm{E})\alpha(\mathrm{F})}}$$

where $\curlywedge(\mathrm{E},\mathrm{F})=\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_E^+(x_i)\,\mu_F^+(x_i)+\mu_E^-(x_i)\mu_F^-(x_i))$,

$\alpha(\mathrm{E})=\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_E^+(x_i))^2+(\mu_E^-(x_i))^2)$ and

$$\alpha(\mathrm{F})=\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_F^+(x_i))^2+(\mu_F^-(x_i))^2).$$

Theorem 9 The correlation coefficient $\theta(\mathrm{E},\mathrm{F})$ satisfies the following properties:

1. $0\le\theta(\mathrm{E},\mathrm{F})\le 1$;
2. $\theta(\mathrm{E},\mathrm{F})=\theta(\mathrm{F},\mathrm{E})$;
3. $\theta(\mathrm{E},\mathrm{F})=1$ if E = F.

Proof: 1. Clearly, $\theta(\mathrm{E},\mathrm{F})\ge 0$.

Let $\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_E^+(x_i))^2=\mathrm{p}$, $\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_F^+(x_i))^2=\mathrm{q}$, $\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_E^-(x_i))^2=\mathrm{r}$, $\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_F^-(x_i))^2=\mathrm{s}$.

It only remains to show that $\theta(\mathrm{E},\mathrm{F})\le 1$. Using the Schwartz inequality, we have

$$\theta(\mathrm{E},\mathrm{F})=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_E^+(x_i)\,\mu_F^+(x_i)+\mu_E^-(x_i)\mu_F^-(x_i))}{\sqrt{\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_E^+(x_i))^2+(\mu_E^-(x_i))^2).\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_F^+(x_i))^2+(\mu_F^-(x_i))^2)}}$$

$$=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_E^+(x_i)\,\mu_F^+(x_i))+\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mu_E^-(x_i)\,\mu_F^-(x_i))}{\sqrt{\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_E^+(x_i))^2+(\mu_E^-(x_i))^2).\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_F^+(x_i))^2+(\mu_F^-(x_i))^2)}}$$

$$\le\frac{\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_E^+(x_i))^2.\sum_{\mathrm{i}=1}^{\mathrm{n}}\left((\mu_F^+(x_i))^2\right]^{\frac{1}{2}}+\left[\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_E^-(x_i))^2.\sum_{\mathrm{i}=1}^{\mathrm{n}}\left((\mu_F^-(x_i))^2\right]^{\frac{1}{2}}}{\sqrt{\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_E^+(x_i))^2+(\mu_E^-(x_i))^2).\sum_{\mathrm{i}=1}^{\mathrm{n}}((\mu_F^+(x_i))^2+(\mu_F^-(x_i))^2)}}$$

$$=\frac{(p.q)^{\frac{1}{2}}+(r.s)^{\frac{1}{2}}}{[(p+r)(q+s)]^{\frac{1}{2}}}$$

As $\theta(\mathrm{E},\mathrm{F})\ge 0$ so the above inequality is equivalent to

$$\theta^2(E,F)\le\frac{pq+2(pqrs)^{\frac{1}{2}}+rs}{(p+r)(q+s)}$$

But $\theta^2(E,F)-1=\frac{pq+2(pqrs)^{\frac{1}{2}}+rs}{(p+r)(q+s)}-1$

$$=-\frac{(pq-2(pqrs)^{\frac{1}{2}}+rs)}{(p+r)(q+s)}$$

$$=-\frac{((ps)^{\frac{1}{2}}-(qr)^{\frac{1}{2}})^2}{(p+r)(q+s)}$$

$$\le 0$$

$\Rightarrow\theta^2(E,F)\le 1$

i.e., $\theta(E,F)\le 1$.

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The proof of 2 and 3 follows from definition directly.

6.Application of proposed bipolar fuzzy entropy

A multi-criteria decision making (MCDM) is the phenomenon of choosing the most favorable alternative which in view of certain criteria. The weight of criterion is very important in the selection of very appropriate alternative. If the evaluation of weights of criteria is not proper, then it may cause to the selecting an inappropriate alternative. Thus the evaluation of weights of criteria is very important and needs careful attention in handling a MCDM problem.

In the following, we discuss the TOPSIS method of MCDM in bipolar fuzzy settings.

TOPSIS is a prevalent technique that offers a reliable solution to a MCDM problem. In this method, the decision is made in view of coefficient of relative closeness depending upon distances of various available options from the ideal solutions. But TOPSIS based upon distance measure however sometimes fails to give the authentic outcomes. The results in such cases be different with different distance measure. Thus, it is required to discover a different TOPSIS method depending on some alternative comparison measure. We present a correlation coefficient base TOPSIS method.

The step wise procedure involved the modified TOPSIS method is given as follows:

Step 1: Setting up bipolar fuzzy decision matrix

Express the input of the MCDM problem using a decision matrix $(M_B)_{m\times n}$ involving bipolar fuzzy values in which rows represents the alternatives namely ξ_i (i=1,2,...,m) and columns depicting criteria namely δ_i(i=1,2,...,n) as under:

$$(M_B)_{m\times n} = \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{m1} & \cdots & \gamma_{mn} \end{pmatrix}$$

Where $\gamma_{11} = (\mu_{ij}^+, \mu_{ij}^-)$ are the bi-polar fuzzy numbers representing the level of satisfaction and counter satisfaction of an option or alternative ξ_i with respect to a criteria δ_j. For computing the degree of satisfaction and counter satisfaction, the method used is analogous to Liu and Wang (2007) as under:

$\mu_{ij}^+ = \frac{\tau(i,j)}{N}$ and $\mu_{ij}^- = \frac{\xi(i,j)}{N}$

where $\tau(i,j)$ determines the number of persons involved in decision making who supports the option or alternative ξ_i with respect to the criteria δ_j and $\xi(i,j)$ denotes that number of persons involved in decision making who opposes the alternative $\xi_i$corresponding to the criteria δ_j and N is the total number of persons involved in making of decision.

Step 2: Formation of normalized decision matrix

Normalize the decision matrix which is obtained in step 1 as follows:

$$(\overline{M_B})_{m\times n} = \begin{pmatrix} \bar{\gamma}_{11} & \cdots & \bar{\gamma}_{1n} \\ \vdots & \ddots & \vdots \\ \bar{\gamma}_{m1} & \cdots & \bar{\gamma}_{mn} \end{pmatrix} \quad (2)$$

where $\bar{\gamma}_{ij} = \begin{cases} (\mu_{ij}^+, \mu_{ij}^-), & forbenefitcriteria \\ (1-\mu_{ij}^+, -1-\mu_{ij}^-), & forcostcriteria \end{cases}$

Step 3: Determination of weights of criteria

Weights of Criteria are very significant in a MCDM problem solution. Thus, for choosing the favourable or acceptable alternative, appropriate assessment of weights of criteria is essential. On the basis of importance of weights of criteria, we divide the process of criteria weights analysis in two segments as under:

(a) ***Case I: When weights of criteria are partially available***

The decision maker's point of view in the solution of a MCDM problem is very useful. If advice of experts is available, it leads to appropriate allocation of weights of criteria. However as a result of few restrictions such as pressure of time, lack of expertise regarding problem domain, etc., there is less possibility of having such advice upon which we can rely. It may leads the experts to present their opinions in the intervals forms despite of exact numbers. The information which is available regarding weights of criteria is called as partial information. We assemble this information available concerning weights of criteria in terms of set namely G. To calculate the weights of criteria, the minimum entropy principle is used as put forward by **Liu and Wang (2007)** as under:

The total entropy of alternatives ξ_i with respect to the criteria δ_j is given by $E_T(\xi_i) = \sum_{j=1}^{n} E_T(\gamma_{ij})$ (3)

where $E_T(\gamma_{ij}) = \frac{1}{n}\sum_{i=1}^{n} \cos\left((\mu_A^+(x_i) + \mu_A^-(x_i))\frac{\pi}{2}\right)$.

Since every alternative displays the fair competition, therefore the coefficients of weights with respect to same criteria should be equal. Thus, we construct a programming model for finding the optimal weights of criteria as under

$$\min(E) = g_j \sum_{i=1}^{m}\sum_{j=1}^{n} \cos\left((\mu_{ij}^+ + \mu_{ij}^-)\frac{\pi}{2}\right) \quad (4)$$

Where $g_j \epsilon G$ provided the condition $\sum_{j=1}^{n} g_j = 1$. After finding the solution of the above linear programming structure, the vector of weights of criteria is given as:

$\arg\min(E) = (g_1, g_2, \dots, g_n)'$, where $'$ represent the transpose.

(b) Case II: When weights of the criteria are not known

Let $C_j^B = \{\bar{\gamma}_{1j}, \bar{\gamma}_{2j}, \dots, \bar{\gamma}_{mj}\}$ be bipolar fuzzy set for criteriaδ_j.

For the determination of weights of criteria, the method used is given by Li (2010) as

$$g_j = \frac{1 - E_T\left(C_j^B\right)}{n - \sum_{j=1}^{n} E_T\left(C_j^B\right)} \quad (5)$$

Step 4: To find bipolar fuzzy positive ideal solution (BFPIS) and bipolar fuzzy negative ideal solution (BFNIS)

We find the positive ideal solution (ξ^+) and negative ideal solution (ξ^-) as follows:

$$\xi^+ = \begin{cases} \left(\sup\left(\mu_{ij}^+\right), \sup\left(\mu_{ij}^-\right)\right) = (1,0) forbenefitcriteria \\ (\inf\left(\mu_{ij}^+\right), inf\,(\mu_{ij}^-)) = (0,-1) \qquad forcostcriteria \end{cases} \quad (6)$$

and

$$\xi^- = \begin{cases} (\inf\left(\mu_{ij}^+\right), inf\,(\mu_{ij}^-)) = (0,-1) \qquad forbenefitcriteria \\ \left(\sup\left(\mu_{ij}^+\right), \sup\left(\mu_{ij}^-\right)\right) = (1,0) forcostcriteria \end{cases} \quad (7)$$

Step 5: Computation of weighted correlation coefficient

Compute the weighted correlation coefficient of alternatives ξ_i $(i = 1,2, \dots, m)$ with $\xi^+$and ξ^-using

$$\theta(\xi_i, \xi^+) = \frac{g_j \curlywedge (\xi_i, \xi^+)}{\sqrt{g_j(\delta(\xi_i)\delta(\xi^+))}} \quad (8)$$

and $$\theta(\xi_i, \xi^-) = \frac{g_j \curlywedge (\xi_i, \xi^-)}{\sqrt{g_j(\delta(\xi_i)\delta(\xi^-))}} \quad (9)$$

Step 6: Computation of relative correlation coefficient

Find the correlation coefficient χ_i $(i = 1,2, \dots, m)$ as under

$$\chi_i = \frac{\theta(\xi_i, \xi^-)}{\theta(\xi_i, \xi^+) + \theta(\xi_i, \xi^-)} \quad (10)$$

Step 7: Decision of best alternative

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Arranging the $\xi_i (i = 1,2, \dots, m)$in accordance with the values of $\chi_i^{\prime} s(i = 1,2, \dots, m)$ in decreasing arrangement, we get a sequence of preference of alternatives.

Illustrative numerical example

Based on the scenario comprehensiveness and feasibility of implementation, a particular fuzzy extension is usually chosen to model a real-system. Before the practical implementation of the proposed algorithm, we justify the choice of bipolar fuzzy set to model the problem under consideration.

We consider a multiple criteria decision making problem whose criterion essentially needs to be given ratings in the bipolar fuzzy framework. We consider the problem of purchasing a suitable COVID-19 vaccine for a target population. In view of very hurried medical inception of the COVID-19 vaccine, some positive as well as negative polarities are associated with the available vaccines in view of certain criteria. Suppose there are five vaccines available prevalently, say, $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$. We consider the four criteria to purchase the medicine, say,δ_1: Cost, δ_2: Effect, δ_3: Availability, δ_4: Public perception.

Among the above mentioned four criteria each one is expected to have positive as well as negative polarity of fuzzy ratings.

Cost: In view of the budget availability, the cost may be given positive rating and it may be given negative rating if government would have to provide subsidies.

Effect: The effect of a medicine is not always positive for every person. Sometimes, it may a cause several complications to the patient. Positive rating is given if the medicine heals the patient and negative polarity is provided if the effect of medicine is negative on the patient, i.e., if the medicine cause several complication to the patient instead of healing.

Availability: In the situation of pandemic like COVID-19, availability of medicines that cures the patient is very important. If the medicine is available within short period of time then a positive rating may be given to the availability criteria and, if the medicine is available on pre-booking and has some waiting time, then the negative rating needs to be given.

Public perception: The perception regarding a medicine in public plays a very important role in vaccination. If their perception is good regarding the medicine, they will not hesitate to take it and if the perception regarding the medicine is negative then they will hesitate to take the medicine or they may not take the medicine. So, positive polarity for positive perception and negative polarity for negative perception of public regarding a medicine is unavoidable. Such a case lead to the consideration of bipolar fuzzy rating for this attribute/criterion.

We investigate the above-mentioned example in two situations:

1) When weights are known partially.

2) When weights are completely unknown.

7.1 When weights are partially known

Example: Consider an organization wants to purchase a suitable COVID-19 vaccine for a target population and there are five vaccines available prevalently, say, $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$. We consider the four criteria to purchase the medicine, say, δ_1: Cost, δ_2: Effect, δ_3: Availability, δ_4: Public perception. The goal is to find the best COVID-19 vaccine.

Depending upon experts' suggestion, the vaccines and their criteria is represented by bipolar fuzzy decision matrix.

Let G be the set denoting the available information regarding the criteria weights and G is given by

$G = \{0.23 \le g_1 \le 0.33, 0.18 \le g_2 \le 0.42, 0.08 \le g_3 \le 0.48, 0.13 \le g_4 \le 0.23\}$ subject to the constraint that $\sum_{j=1}^{4} g_j = 1$.

Applying the proposed MCDM method to choose the best COVID-19 vaccine. The step wise computation are as follows:

Step 1: The MCDM which is given is illustrated by bipolar fuzzy decision matrix $(M_B)_{m \times n}$

in Table 1.

	δ_1	δ_2	δ_3	δ_4
ξ_1	(0.9,-0.2)	(0.6,-0.2)	(0.8,-0.6)	(0.7,-0.6)
ξ_2	(0.7,-0.5)	(0.4,-0.5)	(0.7,-0.2)	(0.4,-0.8)
ξ_3	(0.6,-0.2)	(0.8,-0.4)	(0.5,-0.2)	(0.5,-0.6)
ξ_4	(0.4,-0.5)	(0.9,-0.2)	(0.8,-0.3)	(0.8,-0.2)
ξ_5	(0.8,-0.2)	(0.7,-0.4)	(0.7,-0.1)	(0.7,-0.1)

Table 1. Bipolar fuzzy decision matrix $(\boldsymbol{M_B})_{\boldsymbol{m\times n}}$

***Step 2*:** Since the all criteria involved in the problem are not of same type, so normalization is needed and thus the normalized bipolar fuzzy decision matrix $(\overline{M_B})_{m\times n}$ is given in table 2.

	δ_1	δ_2	δ_3	δ_4
ξ_1	(0.1,-0.8)	(0.6,-0.2)	(0.8,-0.6)	(0.7,-0.6)
ξ_2	(0.3,-0.5)	(0.4,-0.5)	(0.7,-0.2)	(0.4,-0.8)
ξ_3	(0.4,-0.8)	(0.8,-0.4)	(0.5,-0.2)	(0.5,-0.6)
ξ_4	(0.6,-0.5)	(0.9,-0.2)	(0.8,-0.3)	(0.8,-0.2)
ξ_5	(0.2,-0.8)	(0.7,-0.4)	(0.7,-0.1)	(0.7,-0.1)
$E_T(\delta_j)$	0.8003	0.8614	0.8741	0.6635

Table 1. Normalized bipolar fuzzy decision matrix $(\overline{\boldsymbol{M_B}})_{\boldsymbol{m\times n}}$

Step 3: For evaluation of weights of criteria, we follow the following steps:

(i) With the help of proposed bipolar entropy, the calculated numerical values of information for every criteria $\delta_1, \delta_2, \delta_3, \delta_4$ i.e., $E_T(\delta_i), i = 1,2,3,4$; is represented in the bottom of table.

(ii) Using (4), the programming model is as under:

$$minE = 0.8003g_1 + 0.8614g_2 + 0.8714g_3 + 0.66354g_4$$

$$\text{subject to the constraint} \begin{cases} 0.23 \leq g_1 \leq 0.33 \\ 0.18 \leq g_2 \leq 0.42 \\ 0.08 \leq g_3 \leq 0.48 \\ 0.13 \leq g_4 \leq 0.23 \\ \sum_{i=1}^{4} g_i = 1 \end{cases}$$

(iii) On solving this linear programming model using MATLAB, we get the weights of criteria as under:

$$g_1 = 0.33,\ \ g_2 = 0.36,\ \ g_3 = 0.08,\ \ \ g_4 = 0.23$$

Step 4: Using (6) and (7), evaluated values of $\xi^+$and ξ^- are

$\xi^+ = \{(0,-1),(1,0),(1,0),(1,0)\}$

$\xi^- = \{(1,0),(0,-1),(0,-1),(0,-1)\}$

Step 5: The calculated values of weight correlation coefficient using (8) and (9) are

$\theta(\xi_1,\xi^+) = 0.8856, \theta(\xi_2,\xi^+) = 0.6556, \theta(\xi_3,\xi^+) = 0.8366, \theta(\xi_4,\xi^+) = 0.8673,$

$\theta(\xi_5,\xi^+) = 0.9360$

$\theta(\xi_1,\xi^-) = 0.3818, \theta(\xi_2,\xi^-) = 0.6870, \theta(\xi_3,\xi^-) = 0.5083, \theta(\xi_4,\xi^-) = 0.4001,$

$\theta(\xi_5,\xi^-) = 0.3077.$

Step 6: The calculated values of relative correlation coefficient$\chi_i{}'s(i = 1,2,\ldots,5)$ are

$\chi_1 = 0.3012,\ \ \chi_2 = 0.5116,\ \ \ \chi_3 = 0.3779,\ \ \chi_4 = 0.3156,\ \ \ \chi_5 = 0.2431.$

Step 7: Arranging the $\xi_i(i = 1,2,\ldots,m)$in accordance with the values of $\chi_i's(i = 1,2,\ldots,m)$ in decreasing order, we get

$$\xi_2 > \xi_3 > \xi_4 > \xi_1 > \xi_5$$

Therefore, using the partially known criteria weights, we find that vaccine ξ_2 is best one in this case.

7.2 When the weights of criteria are not known

We consider the above example for completely unknown criteria-weights. Following are the steps in computation:

Step 1: The criteria weight calculated using (5) are

$$g_1 = 0.2416,\ \ g_2 = 0.2556,\ \ g_3 = 0.0.2463,\ \ g_4 = 0.2563$$

Step 2: Using evaluated values of $\xi^+$and $\xi^-$are

$\xi^+ = \{(0,-1),(1,0),(1,0),(1,0)\},$

$\xi^- = \{(1,0),(0,-1),(0,-1),(0,-1)\}.$

Step 3: The calculated values of weight correlation coefficient using (8) and (9) are

$\theta(\xi_1,\xi^+) = 0.8500, \theta(\xi_2,\xi^+) = 0.6876, \theta(\xi_3,\xi^+) = 0.8206, \theta(\xi_4,\xi^+) = 0.8871,$

$\theta(\xi_5,\xi^+) = 0.9493;$

$\theta(\xi_1,\xi^-) = 0.4430, \theta(\xi_2,\xi^-) = 0.3285, \theta(\xi_3,\xi^-) = 0.5082, \theta(\xi_4,\xi^-) = 0.3789,$

$\theta(\xi_5,\xi^-) = 0.2638.$

Step 4: The calculated values of relative correlation coefficient χ_i's (i=1,2,3,4,5) are

$\chi_1 = 0.3426,\ \ \chi_2 = 0.3232,\ \ \ \chi_3 = 0.3824,\ \ \chi_4 = 0.2992,\ \ \ \chi_5 = 0.2171.$

Step 5: Arranging the $\xi_i(i = 1,2,\ldots,m)$in accordance with the values of $\chi_i's(i = 1,2,\ldots,m)$ in decreasing order, we get

$$\xi_3 > \xi_1 > \xi_2 > \xi_4 > \xi_5$$

Thus, ξ_3COVID-19 vaccine is found to be best one when no information is available about the weights of criteria.

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Conclusion

In this paper some underlying notions like crispness, most fuzzy state and relative fuzziness (one set is less fuzzy than other) in reference to bipolar fuzzy logic lead to develop an axiomatic framework for defining a bipolar fuzzy entropy. Some novel operations along with some properties has been studied on a bipolar fuzzy set A. A Characterization theorem has also been proved to obtain a class of bipolar fuzzy entropy measures. Further three newly introduced bipolar fuzzy entropy measures have been axiomatically validated. A newly proposed bipolar fuzzy correlation coefficient and the proposed bipolar fuzzy entropy have been utilized to introduce a novel algorithm for MADM problems with partially known and completely unknown attribute weights.

Compliance with ethical standards

Ethical approval : The present article does not contain any studies with human participants or animals performed by any of the authors.

Data availability: This paper utilizes artificial data and can be referred directly from the paper.

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