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Sombor and Euclidean Reciprocal Index of Subdivision of Some Networks

[1]Akhshaya P S, [2]S. S. Sandhya

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Abstract: A topological index is a numerical quantity computed from the molecular graph of a compound, reflecting structural characteristics such as branching, symmetry, and connectivity. Such indices are widely used in QSAR and QSPR modeling to estimate physicochemical and biological properties without extensive experimental work. In this paper find, the Sombor and Euclidean Reciprocal Index (ERI) of some subdivision networks.

Keywords *- Graph, Oxide Network, Rhombus Oxide Network, Silicate Network, Chain Silicate Network, Rhombus Silicate Network.*

Subject Classification: 05C09, 05C92, 05C07.

Introduction

This paper deals solely with finite, simple, undirected graphs. Let $G = (V, E)$ be such a graph, having p vertices and q edges. Standard terminology and notation follow Harary [3]. Topological indices are numerical graph invariants that encode structural properties, serving a powerful tool for analyzing molecular behavior, network robustness, and biological interactions.

The field traces its origins to the Wiener index (1947), which correlated hydrocarbon structures with physical properties. This was followed by the Randić index in 1975, introduced to model molecular branching and heteroatom effects. A landmark advancement occurred in 1972 when Gutman and Trinajstić jointly introduced the first and second Zagreb indices, which have since become foundational benchmarks in mathematical chemistry

Most recently, Gutman (2021) introduced the Sombor index, a geometrically inspired invariant based on the Euclidean norm of the degree pairs of adjacent vertices, which has garnered substantial attention for its exceptional predictive capacity [2]:

$$SO(G) = \sum_{uv \in E} \sqrt{d(u)^2 + d(v)^2}$$

[1]*Reg No.: 23213182092001*

Department of Mathematics, Sree Ayyappa College for Women,

Chunkankadai, Tamilnadu, India.

[Affiliated to Manonmaniam Sundaranar University, Abhishekapatti, Thirunelveli-627012]

Email: akhshayapadmakumar12@gmail.com

[2]*Department of Mathematics, Sree Ayyappa College for Women,*

Chunkankadai, Tamilnadu, India.

[Affiliated to Manonmaniam Sundaranar University, Abhishekapatti, Thirunelveli-627012]

Email: sssandhya2009 @gmail.com

where $d(u)$ and $d(v)$ are the degrees of adjacent vertices u and v.

Inspired by the mathematical elegance and practical utility of Gutman's Sombor index (SO), we propose the Euclidean Reciprocal Index (ERI) as a novel advancement in graph-theoretical characterization. The ERI of a graph G is formally defined as the sum over all edges $uv \in E(G)$, of the square roots of the reciprocal squared degrees of adjacent vertices

$$\mathrm{ERI(G)} = \sum_{\mathrm{uv}\in \mathrm{E(G)}} \sqrt{\frac{1}{\mathrm{d(u)}^2} + \frac{1}{\mathrm{d(v)}^2}}$$

Where $\mathrm{E(G)}$ is the edge set of G, $\mathrm{d(u)}$ and $\mathrm{d(v)}$ denote the degrees of adjacent vertices u and v, respectively.

Here we calculate the subdivision of networks, The subdivision graph $S(G)$ of a graph G is obtained by inserting a new vertex of degree 2 into every edge of G, so that each edge uv is replaced by a path $u - w - v$. If G has n vertices and m edges, then $S(G)$ $has\ n + m$ vertices and $2m$ edges. In $S(G)$ the original vertices keep their degrees, and every edge joins a vertex of degree d_v to a vertex of degree 2. Consequently, every edge of $S(G)$ is of the form $(d_v, 2)$, and the edge partition of $S(G)$ is determined by the vertex degrees of G.

In this paper we compute SO and ERI for the subdivision graphs of the oxide, silicate, chain silicate, rhombus oxide and rhombus silicate networks.

1. Main Results:

Theorem: 2.1.

For $n \geq 1$, $SO\big(S(OX_n)\big) = 72\sqrt{5}n^2 + 24\big(\sqrt{2} - \sqrt{5}\big)n$

$ERI\big(S(OX_n)\big) = 3\sqrt{5}n^2 + n\big(6\sqrt{2} - \sqrt{5}\big)$

Proof.

Let $u_1, u_2, \ldots, u_n$ be vertices of OX_n

Oxide network has $9n^2 + 3n$ vertices and $18n^2$ edges. There are $6n$ vertices of degree 2 and $9n^2 - 3n$ vertices of degree 4.

Table 1. Edge Partition of Subdivision of Oxide Network based on degree of vertices

(d_u, d_v)	Number of adjacent pair of vertices u, v
(2,2)	$12n$
(2,4)	$36n^2 - 12n$

Sombor index of subdivision oxide network is given by

$$SO\big(S(OX_n)\big) = 12n\sqrt{2^2 + 2^2} + (36n^2 - 12n)\sqrt{2^2 + 4^2}$$

$$= 24\sqrt{2}n + 72\sqrt{5}n^2 - 24\sqrt{5}n$$
$$= 72\sqrt{5}n^2 + 24\big(\sqrt{2} - \sqrt{5}\big)n$$

Euclidian Reciprocal index of subdivision oxide network is given by

$$ERI\big(S(OX_n)\big) = 12\mathrm{n}\sqrt{\tfrac{1}{2^2} + \tfrac{1}{2^2}} + (36n^2 - 12n)\sqrt{\tfrac{1}{2^2} + \tfrac{1}{4^2}}$$

$$= \frac{12n}{\sqrt{2}} + \frac{12n(3n-1)\sqrt{5}}{4}$$
$$= 3\sqrt{5}n^2 + n\big(6\sqrt{2} - \sqrt{5}\big)$$

Example 2.1. Figure 2.1 shows OX_5, $SO\big(S(OX_5)\big) = 3926.3$, $ERI\big(S(OX_5)\big) = 198.9$

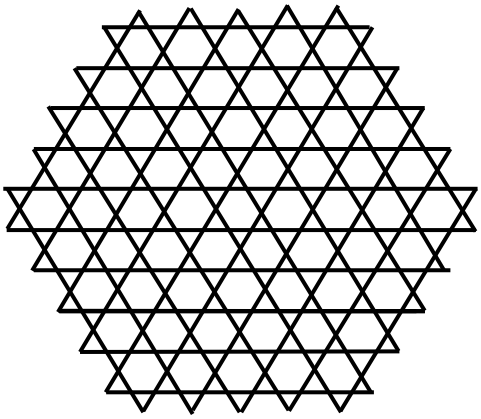


Figure 2.16. Oxide Network

Theorem: 2.2.

For $n \geq 1$, $SO\big(S(OX_n)\big) = 24\sqrt{5}n^2 + 16\big(\sqrt{2} - \sqrt{5}\big)n$

$ERI\big(S(OX_n)\big) = 3\sqrt{5}n^2 + 2\big(2\sqrt{2} - \sqrt{5}\big)n$

Proof.

Let $u_1, u_2, \dots, u_n$ be vertices of OX_n. Oxide Network has $3n^2 + 2n$ vertices and $6n^2$ edges. There are $4n$ vertices of degree 2 and $3n^2 - 2n$ vertices of degree 4.

Table 2. Edge Partition of Subdivision of rhombus oxide network ($RHOX_n$) based on degree of vertices

(d_u, d_v)	Number of adjacent pair of vertices u, v
(2,2)	$8n$
(2,4)	$12n^2 - 8n$

Sombor index of subdivision of rhombus oxide network is given by

$$SO\big(S(RHOX_n)\big) = 8n\sqrt{2^2 + 2^2} + (12n^2 - 8n)\sqrt{2^2 + 4^2}$$

$$= 16\sqrt{2}n + 24\sqrt{5}n^2 - 16\sqrt{5}n$$
$$= 24\sqrt{5}n^2 + 16\big(\sqrt{2} - \sqrt{5}\big)n$$

Euclidian Reciprocal index of subdivision of rhombus oxide network is given by

$$ERI\big(S(RHOX_n)\big) = 8n\sqrt{\frac{1}{2^2} + \frac{1}{2^2}} + (12n^2 - 8n)\sqrt{\frac{1}{2^2} + \frac{1}{4^2}}$$

$$= 4n\sqrt{2} + \frac{(12n^2 - 8n)\sqrt{5}}{4}$$
$$= 3\sqrt{5}n^2 + 2\big(2\sqrt{2} - \sqrt{5}\big)n$$

Example 2.2. Figure 2.2 shows $RHOX_3$, $SO(RHOX_3) = 443.5$, $ERI(RHOX_3) = 63.9$

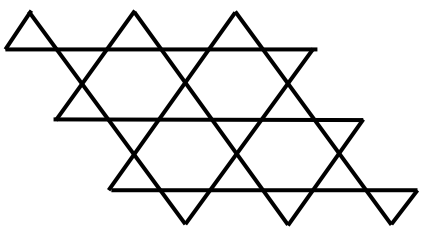


Figure 2.2. Rhombus Oxide Network

Theorem: 2.3.

For $n \geq 1$, $SO\big(S(SL_n)\big) = 18n((n+1)\sqrt{13} + 2(3n-1)\sqrt{10}$

$ERI\big(S(SL_n)\big) = 3n\left((n+1)\sqrt{13} + (3n-1)\sqrt{10}\right)$

Proof.

Silicate network has $15n^2 + 3n$ vertices and $36n^2$ edges**.** There are $6n^2 + 6n$ vertices of degree 3 and $9n^2 - 3n$ vertices of degree 6

Table 3. Edge Partition of Subdivision of Silicate network (SL_n) based on degree of vertices

(d_u, d_v)	Number of adjacent pair of vertices u, v
(2,3)	$18n(n+1)$
(2,6)	$18n(3n-1)$

Sombor index of subdivision of silicate network is given by

$$SO\big(S(SL_n)\big) = \big(18n(n+1)\big)\sqrt{2^2+3^2} + \big(18n(3n-1)\big)\sqrt{2^2+6^2}$$
$$= \big(18n(n+1)\big)\sqrt{13} + \big(18n(3n-1)\big)\sqrt{40}$$
$$= 18n((n+1)\sqrt{13} + 2(3n-1)\sqrt{10}$$

Euclidian Reciprocal index of subdivision of silicate network is given by

$$ERI\big(S(SL_n)\big) = \big(18n(n+1)\big)\sqrt{\frac{1}{2^2}+\frac{1}{3^2}} + \big(18n(3n-1)\big)\sqrt{\frac{1}{2^2}+\frac{1}{6^2}}$$
$$= \frac{18n(n+1)\sqrt{13}}{6} + \frac{\big(18n(3n-1)\big)2\sqrt{10}}{12}$$
$$= 3n\left((n+1)\sqrt{13} + (3n-1)\sqrt{10}\right)$$

Example 2.3. Figure 2.3 shows SL_2, $SO(SL_2) = 1527.8$, $ERI(SL_2) = 159.77$

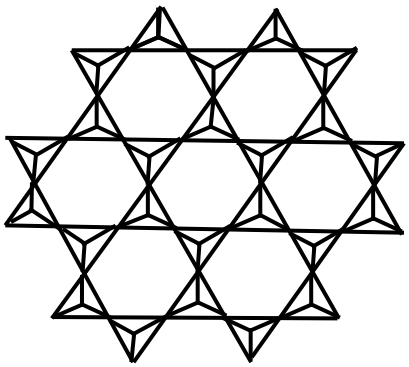


Figure 2.3. Silicate Network

Theorem: 2.4.

For $n \geq 1$, $SO\big(S(CSL_n)\big) = 6\left((n+1)\sqrt{13} + 2(n-1)\sqrt{10}\right)$

$ERI\big(S(CSL_n)\big) = (n+1)\sqrt{13} + (n-1)\sqrt{10}$

Proof.

Chain silicate network has $3n+1$ vertices and $6n$ edges. There are $n+4$ vertices of degree 3 and $2n-3$ vertices of degree 6.

Sombor coindex of chain silicate network CSL_n is given by

Table 4. Edge Partition of Subdivision of Chain Silicate network (CSL_n) based on degree of vertices

(d_u, d_v)	Number of adjacent pair of vertices u, v
(2,3)	$6(n+1)$
(2,6)	$6(n-1)$

Sombor index of subdivision of chain silicate network is given by

$$SO\big(S(CSL_n)\big) = \big(6(n+1)\big)\sqrt{2^2+3^2} + \big(6(n-1)\big)\sqrt{2^2+6^2}$$
$$= \big(6(n+1)\big)\sqrt{13} + \big(6(n-1)\big)\sqrt{40}$$
$$= 6\Big((n+1)\sqrt{13} + 2(n-1)\sqrt{10}\Big)$$

Euclidian Reciprocal index of subdivision of chain silicate network is given by

$$ERI\big(S(CSL_n)\big) = \big(6(n+1)\big)\sqrt{\tfrac{1}{2^2}+\tfrac{1}{3^2}} + \big(6(n-1)\big)\sqrt{\tfrac{1}{2^2}+\tfrac{1}{6^2}}$$
$$= \frac{6(n+1)\sqrt{13}}{6} + \frac{\big(6(n-1)\big)2\sqrt{10}}{12}$$
$$= (n+1)\sqrt{13} + (n-1)\sqrt{10}$$

Example 2.4. Figure 2.4 shows CSL_5, $SO(CSL_5) = 281.58$, $ERI(CSL_5) = 34.28$

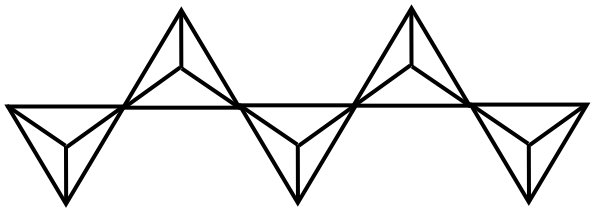


Figure 2.4. Chain Silicate Network

Theorem: 2.5.

For $n \geq 1$, $SO\big(S(RHSX_n)\big) = 2(3n^2+6n+2)\sqrt{13} + \big(12n(3n-2)\big)\sqrt{10}$

$$ERI\big(S(RHSX_n)\big) = \frac{(3n^2+6n+2)\sqrt{13}}{3} + n(3n-2)\sqrt{10}$$

Proof.

Let $u_1, u_2, \ldots, u_n$ be vertices of SX_n. Rhombus Silicate Network has $5n^2+2n$ vertices and $12n^2$ edges**.** There are $2n^2+4n$ vertices of degree 3 and $3n^2-2n$ vertices of degree 6.

Sombor coindex of rhombus silicate network $RHSX_n$ is given by

Table 5. Edge Partition of Subdivision of Rhombus Silicate network ($RHSX_n$) based on degree of vertices

(d_u, d_v)	Number of adjacent pair of vertices u, v
(2,3)	$6n^2+12n+4$
(2,6)	$6n(3n-2)$

Sombor index of subdivision of rhombus silicate network is given by

$$SO\big(S(RHSX_n)\big) = (6n^2+12n+4)\sqrt{2^2+3^2} + (6n(3n-2))\sqrt{2^2+6^2}$$
$$= 2(3n^2+6n+2)\sqrt{13} + \big(12n(3n-2)\big)\sqrt{10}$$

Euclidian Reciprocal index of subdivision of rhomnus silicate network is given by

$$ERI\big(S(RHSX_n)\big) = (6n^2 + 12n + 4)\sqrt{\tfrac{1}{2^2} + \tfrac{1}{3^2}} + (6n(3n-2))\sqrt{\tfrac{1}{2^2} + \tfrac{1}{6^2}}$$
$$= \frac{2(3n^2 + 6n + 2)\sqrt{13}}{6} + \frac{(6n(3n-2))2\sqrt{10}}{12}$$
$$= \frac{(3n^2 + 6n + 2)\sqrt{13}}{3} + n(3n-2)\sqrt{10}$$

Example 2.5. Figure 2.5 shows $RHSX_3$, $SO(RHSX_3) = 1135.8,\ ERI(RHSX_3) = 122.9$

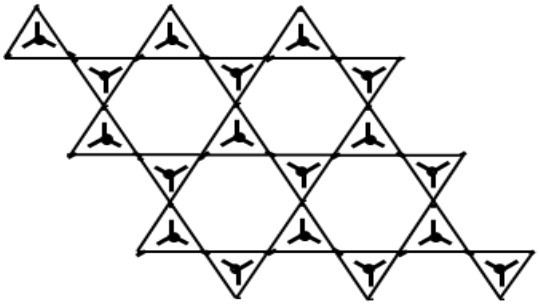


Figure 2.5. Rhombus Silicate Network

Conclusion

In this paper, we find Sombor and Euclidean Reciprocal Index of some subdivision networks like Oxide Network, Rhombus Oxide Network, Silicate Network, Chain Silicate Network, and Rhombus Silicate Network. Future research will focus on expanding ERI's applications to dynamic network analysis, biological systems, and machine learning approaches for materials discovery. We anticipate significant potential in developing weighted ERI variants for complex systems and establishing theoretical bounds for extremal graphs.

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